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INTELLECTUAL PROCESSES RELATED TO MATHEMATICS ACHIEVEMENT AT
GRADE LEVELS 4, 5, AND 6.

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RELATIONSHIPS BETWEEN CERTAIN INTELLECTUAL PROCESSES AND
ACHIEVEMENT OF INSTRUCTIONAL OBJECTIVES FOR SELECTED
MATHEMATICS TOPICS TAUGHT IN THE HIGHER ELEMENTARY GRADES
WERE EXAMINED. INSTRUMENTS WERE CHOSEN TO ASSESS MATHEMATICS
ACHIEVEMENT AND INTELLECTUAL PROCESSES. THESE WERE
ADMINISTERED BY CLASSROOM TEACHERS TO 765 STUDENTS IN THE
FOURTH, FIFTH, AND SIXTH GRADES WHO HAD SATISFIED SPECIFIC
SELECTION CRITERIA. MATHEMATICS ACHIEVEMENT DATA WERE
GATHERED ON (1) NUMERICAL OPERATIONS AND ASSUMPTIONS, (2)
GEOMETRY, (3) RELATIONS, (4) NUMERATION, (5) MEASUREMENT, AND
(6) OPERATIONS WITH FRACTIONS. DATA GATHERED ON INTELLECTUAL
PROCESSES WERE (1) PERCEPTUAL DISCRIMINATION, (2) MEMORY
SPAN, (3) ASSOCIATIVE MEMORY, (4) ASSOCIATIVE MEMORY PLUS
CONVERSION, (5) REASONING ON NUMERICAL RELATIONS AND WORD
GROUPINGS, (6) VERBAL MEANING, AND (7) SPATIAL RELATIONS. A
SINGLE CLASSIFICATION ANALYSIS OF VARIANCE WAS USED TO
ANALYZE RELATIONSHIPS AMONG THE THREE GRADES' MEANS. THE
RESULTS OF THE STUDY INDICATED THAT THE PREDOMINANT
PSYCHOLOGICAL COMPONENTS (INTELLECTUAL PROCESSES) OF THE
FACETS OF MATHEMATICAL ACHIEVEMENT SAMPLED WERE FACILITY IN
NUMERICAL REASONING AND IN DISCERNING VERBAL MEANING. RESULTS
WERE CONSISTENT WITH DATA REPORTED IN COMPARABLE
INVESTIGATIONS CONDUCTED EARLIER. (JH)

U. S. DEPARTMENT OF HEALTH, EDUCATION AND WELFARE
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**INTELLECTUAL PROCESSES RELATED TO MATHEMATICS ACHIEVEMENT
AT GRADE LEVELS FOUR, FIVE, AND SIX**

Cooperative Research Project No. S-046

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**with the cooperation of
Norma W. Galbreath**

**Office of General Research, University of Georgia
and the
Clarke County (Georgia) School District**

**Athens, Georgia
1965**

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the Cooperative Research Program of the Office of Educa-
tion, U. S. Department of Health, Education, and Welfare.**

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Conducting and reporting an investigation such as this one involves the participation of many people. It is a pleasure to acknowledge and express appreciation to those named below. Their cooperation and assistance contributed immeasurably to the completion of the research project.

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Mrs. Norma Galbreath coordinated and supervised activities involved in the preparation of the research materials and the final report. Mr. Robert M. Brown and Mr. L. Wayne Swindel decoded data after machine processing and executed additional statistical analyses. Mrs. Elizabeth Bryan was responsible for the computer operations. Personnel who scored research instruments and encoded data were Miss Joan Bond, Mr. Michael Bradley, and Miss Sharon Riser. Miss Malese Anderson, Mrs. Margaret Jones, and Mrs. Lynn Monroe assisted with the typing and proofreading of the final manuscript; also, they collated and checked material for the final report.

The research project was conducted with the cooperation of the Clarke County (Georgia) School District. Many public school pupils participated in the data collection phase of the project. The following are cooperating public school personnel who granted permission

for the research to be conducted with pupils under their jurisdiction: Mr. Sam W. Wood, Superintendent; Mr. Alton A. Ellis, Associate Superintendent; and Dr. J. C. Mullis, Testing and Guidance Director. Appendix A contains the names of the principals of the schools from which pupils were selected and the names of the teachers who administered the tests to the subjects.

Appreciation is extended to Educational Testing Service and to the California Test Bureau for granting permission to use selected test items in the research instruments.

While those who were named in the prefatory statement made valuable contributions to the research project, the project directors alone bear the entire and final responsibility for any errors and inadequacies in planning, execution, interpretation, and reporting.

H. W.

K. B.

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CHAPTER I

PROBLEM

Purpose and Objectives

The purpose of this study was to examine relationships between certain intellectual processes and achievement of instructional objectives for selected mathematics topics and subtopics taught at grade levels four, five and six. The collection of evidence relevant to this purpose was guided by four specific research objectives.

1. To identify intellectual processes which are related, singly or in combination, to achievement of instructional objectives for each mathematics topic and subtopic.
2. To determine to what extent the identified intellectual processes or combinations of intellectual processes are related to achievement of instructional objectives for each mathematics topic and subtopic.
3. To specify the relative contribution, or weighting, of each intellectual process when combinations of intellectual processes are related to achievement of instructional objectives for each mathematics topic and subtopic.
4. To determine whether intellectual processes related to achievement differ among mathematics topics and subtopics.

Educational Significance

Essentially, the interest in this study was to obtain information which could be helpful in specifying a relational bridge between two domains: (1) mathematics education and (2) psychological research on variables affecting learning and on the nature of intelligence. Information obtained in a study like this could be useful to those responsible for planning and conducting applied research projects and development projects related to various educational problems. An example of such research and development activities to which the results of the present study could be pertinent is an endeavor like the following: translating implications of research data on learning and intelligence into specific differential procedures for teaching mathematics to intellectually retarded, normal, and superior elementary school pupils.

In writing individually or in summarizing the deliberations of

groups, several authors¹ have identified the need to apply psychological knowledge and research techniques directly in the solution of educational problems; in their presentations, these authors have emphasized the cognitive domain. Hall and Clark (1960, p. iii) related this need to mathematics education.

It is becoming increasingly clear that training in mathematics is a key to continuance of scientific and technological progress. As a result, various projects for the improvement of teaching mathematics have been established throughout the country. ...These groups have concentrated mainly on the rewriting of textbook materials and the use of new principles of teaching. ...The contributions which psychology can make to the improvement of mathematics teaching have not as yet been fully assessed or exploited.

Numerous proposals² have been made about problems in mathematics education to which research related to learning and the nature of intelligence is pertinent. Also, much information³ has been accumulated in these two areas of psychological research. However, a subtle problem arises in the process of translating such research into procedures for mathematics education: i.e., practical experience and intensive analysis have revealed that there is still lacking in many cases, what Coladarci (1963, p. 3) termed "heuristic bridges between the domain of pedagogy and the ... science of psychology." That is, mathematics achievement requires complex behavior which might be expected to involve a number of more unitary, underlying processes (Sells, 1960a, 1960b). Identifying psychological components which underlie mathematics achievement could be a helpful preparatory step

¹For example, see the materials presented by Clark and Fehr (1953), DuBois and Feierabend (1960), Forehand (1962), Johnson (1957), McKeachie (1962), and Underwood, et.al. (1958).

²For example, see the problems and research designs presented by Brown and Kinsella (1960), Clark and Fehr (1953), DuBois (1960a, 1960b), Gagne (1960a, 1960b), Henderson (1963), Manning (1960), and Sells (1960a, 1960b).

³For example, see materials presented by Anderson and Ausubel (1965), Blake (1959, 1961, 1964), Cruickshank (1946), Deese (1958), Feierabend (1960), Gagne (1965), Guilford (1959), Harper, Anderson, Christensen, and Hunka (1964), Johnson and Blake (1960), McGeoch and Irion (1952), Munn (1954), Russell (1956), Travers (1963), Tucker (1960), and Underwood (1949).

in the process of using psychological research as a partial basis for decision making about specific procedures for mathematics education. The importance of this preparatory step was discussed by Underwood, Beberman, Gagne, Kendler, Melton, Russell, and Spence (1958, p. 8) who served as a panel to consider the pertinence of research on fundamental learning processes to educational practice. Among their comments was the following statement.

It is probably fair to say that the panel roamed rather widely over and around topics or ideas or principles of learning that were believed to have at least potential application in our educational systems. In the discussion of possible applications, one matter rather persistently inserted itself. This has to do with the necessity for an analysis of the subjects or tasks that are taught in our schools. By this is meant that the panel felt that a cardinal need was a breaking down of these tasks or subject matters so that they become more understandable in a research sense. Insofar as possible, subject matters should be broken down into components for research purposes. Although the exact nature of the breakdown cannot be specified, the learning of a given subject, when analyzed, might be shown to be a certain part rote learning, a certain part concept formation, a certain part problem solving, etc. Research attacks might well be expected to vary depending upon the results of such analysis.

The present study involved applying the recommendation by Underwood, et.al. to content of mathematics education; that is, this study was an attempt to identify some of the intellectual components of selected aspects of mathematics achievement. Evidence yielded by the study could contribute to the identification of a relational bridge between parts of the domains of psychology and mathematics education. And so, this evidence and similar evidence obtained in subsequent studies could be useful in applying specific data accumulated in psychological research on learning and intelligence to particular mathematics education procedures. For this reason, the study was considered to be educationally significant.

Related Research

Research considered relevant to this study was restricted to empirical investigations of how various intellectual processes are related to mathematics achievement. Twelve investigations contained data which are pertinent, to some extent, to the research problem of the present study. These researchers were Barrett (1952), Blackwell

(1940), Chase (1958), Corle (1958), Hansen (1943), Holzinger and Swineford (1946), Lee (1955), Murray (1949), Olckers (1951), Shine (1960), Stake (1958), and Sutherland (1942). Among aspects of mathematics considered by these investigators were topics in arithmetic, algebra, geometry, and trigonometry. In the various studies, subjects were selected from all school levels -- elementary, high school, and college. Intellectual processes studied included variables like perception of spatial relationships, reasoning, verbal comprehension, word fluency, and memory.

The evidence obtained by the investigators cited above represented a valuable contribution to identifying intellectual components of mathematics achievement. However, generalization was limited for at least two reasons.

1. The several investigations involved diverse aspects of mathematics, pupils at various educational levels, and limited numbers of intellectual processes.
2. Behavior considered by the different investigators did not include sufficiently extensive samples of either mathematics achievement or intellectual processes.

A need existed for an extensive and systematic study in which a number of different aspects of mathematics and intellectual processes were examined with a constant group of subjects which included samples from several grade levels. Such a study could yield information about relationships between a variety of intellectual processes and aspects of mathematics achievement. The present study was concerned with this need.

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CHAPTER II

PROCEDURE

Research Design

The research design illustrated in Figure 1 was used for obtaining data pertinent to relationships between intellectual processes and mathematics achievement. One set of responses, designated Y-variables, was the subjects' scores on instruments chosen to assess achievement of selected mathematics instructional objectives. The other set of responses, designated X-variables, consisted of the subjects' scores on instruments designed to assess certain intellectual processes. The design was replicated with each subject.

Briefly, the procedures used were as follows. Classes of fourth, fifth, and sixth grade pupils were administered mathematics achievement tests and intellectual processes tests. The tests were administered by classroom teachers. These procedures are described more specifically below in terms of subjects, instruments for assessing mathematics achievement, instruments for assessing intellectual processes, general procedures, and analyses.

Subjects

The subjects were 244 fourth grade pupils, 261 fifth grade pupils, and 260 sixth grade pupils. These pupils were selected from the total number of pupils enrolled at grade levels four, five, and six in the Clarke County (Georgia) School District during the 1963-1964 school year. The number and sizes of the classes sampled and steps involved in the sampling process are specified below.

The total enrollment at each grade level consisted of approximately 500 pupils. These pupils attended eight elementary schools; each elementary school contained either two or three classes at each grade level. Class size ranged from 25 to 40 pupils.

The first step in the sampling process was to select classes of pupils at each grade level. The selection of classes was based primarily on two criteria. The first criterion was that the classroom teacher had taught in the elementary schools in Clarke County School

Figure 1

Research Design

Scores Reflecting Intellectual Processes (X)	Scores Reflecting Mathematics Achievement (Y)						
	Y1	Y2	Y3	Y4	Y5	...	Yn
X1							
X2							
X3							
X4							
X5							
⋮							
Xn							

District for at least the previous three years. The second criterion was that the classroom teacher had been an active participant in the School District's in-service mathematics education program.

These two class selection criteria were used to insure that teachers of the prospective subjects had had training and experience in teaching mathematics as presented in Volumes 1, 2, and 3 of the state curriculum guide, Mathematics for Georgia Schools¹ (Georgia Curriculum Guide Committee, 1962). The Clarke County School District has been in the process of revising its mathematics program for the past few years, using the state curriculum guide as a basic framework to improve mathematics instruction in the local schools. The revision process has included various activities² designed to increase the effectiveness with which teachers teach mathematics.

¹This curriculum guide was developed, tried-out, and revised over a period of several years. It was designed to serve as a framework for teaching mathematics in the Georgia Schools. Members of the Georgia Mathematics Curriculum Guide Committee were personnel from throughout the state including the following: from the Georgia State Department of Education and the universities--mathematicians and mathematics educators; from the public school systems--instructional supervisors, principals, secondary school mathematics teachers, and elementary school teachers who had special training and competence in mathematics education. Consultants to the Committee were a number of mathematicians and mathematics educators from universities throughout the nation and from federal and private agencies.

²Some of the activities included in this revision process were the following:

- a. In-service teacher education programs were instituted; these included intensive work during the school year and during pre-school and post-school planning periods. Serving these in-service programs as consultants were mathematicians and mathematics educators from throughout the nation, the Georgia State Department of Education, and the University of Georgia.
- b. Mathematics courses for teachers were taught by University of Georgia professors. The expenses incurred by teachers who participated in these courses were borne by the Clarke County Board of Education.
- c. Closed circuit T.V. programs for teachers were presented; in these T.V. programs, master teachers in the Clarke County School District demonstrated methods for teaching modern mathematics.

Twenty-six classes were yielded by application of the class selection criteria related to length of the teacher's service and to the teacher's participation in mathematics education activities. There were 9 classes on the fourth grade level, 8 classes on the fifth grade level, and 9 classes on the sixth grade level. Figure 2 contains data reflecting the distribution of these classes by schools. Appendix A consists of a list of school personnel who participated in the data collection procedures.

The second step in the sampling process was to identify pupils who were eligible to participate as subjects. Eligible to be included as subjects were those pupils in the selected classes who were taught in the mathematics instructional program during the 1963-1964 school year and who satisfied the following additional selection criteria.

1. Absence of gross physical or sensory impairments which might interfere with responses to the mathematics instructional program or the instruments used in the study.
2. Absence of major emotional and motivational difficulties which might interfere with responses to the mathematics instructional program or the instruments used in the study.

Contacts were made with principals and teachers in order to identify subjects who did not meet the selection criteria. Pupils who did not meet the selection criteria were identified and designated for exclusion from the study. However, for humane and practical reasons, the tests were administered to the excluded pupils at the same time they were administered to the other pupils in the classes. After the tests were collected, the excluded pupils' tests were removed. The excluded pupils' scores were not tabulated nor included in the data analyses. The number of pupils who qualified as subjects was as follows: fourth grade -- 244, fifth grade -- 261, sixth grade -- 260.

Instruments for Assessing Mathematics Achievement

Definitions of Terms

The mathematics achievement data were interval scores on instruments selected to reflect the extent to which the pupils attained instructional objectives for mathematics topics and subtopics which they were taught specifically. Listed below are specifications of

Figure 2

Number of Classes Selected From Total Number of Available
Classes Categorized by School and Grade Level

School	4th Grade		5th Grade		6th Grade	
	Total Number of Classes	Number of Classes Selected	Total Number of Classes	Number of Classes Selected	Total Number of Classes	Number of Classes Selected
1	4	2	3	2	3	2
2	2	1	2	1	2	1
3	3	2	3	2	2	2
4	1	1	1	0	1	1
5	2	1	2	1	2	1
6	1	0	1	1	1	1
7	2	1	2	1	2	1
8	1	1	1	0	1	0
Total	16	9	15	8	14	9

¹ terms pertinent to the mathematics behavior sampled.

- a. Mathematics content. The domain or universe of mathematics taught to pupils at any one grade level.
- b. Mathematics topics. Areas of mathematics content, for example: number, operation, assumptions; numeration.
- c. Mathematics subtopics. Specific subdivisions of content within the mathematics topics. For example, the topic--number, operation, assumptions--includes such subtopics as these: Addition, Multiplication.
- d. Mathematics instructional objectives. Precise specifications of behavior relevant to the subtopics to be attained through the instructional program, for example: understanding the concept of rational numbers, acquiring facility in adding two-place numbers.

Description of the Test

Generally, the selection of test items to be included in the criterion measure was limited to materials sampling the instructional objectives of the mathematics curriculum for the fourth, fifth, and sixth grades. Specifically, the test was designed to measure achievement of selected instructional objectives relevant to six topics, each of which was divided into several subtopics. Each subtopic was represented by four test items. The composition of the test is portrayed in Figure 3. This figure contains the names of the topics and subtopics and the number of test items relevant to each.

The framework of the mathematics curriculum to which the subjects were exposed was the Georgia mathematics curriculum guide, Mathematics for Georgia Schools (Georgia Curriculum Guide Committee, 1962). This guide was considered to be a task analysis of the curriculum as it is taught in the Clarke County School District. The guide consists of topics, subtopics, instructional objectives, and specific teaching activities. Developing the mathematics test involved the following steps.

1. Reviewing in detail the curriculum guide, Mathematics for Georgia Schools.
2. Selecting six topics based on materials presented in the curriculum guide. Designating several subtopics relevant to each topic. For these topics and subtopics, further

¹ These specifications are based on materials presented in Mathematics for Georgia Schools, (Georgia Curriculum Guide Committee, 1962).

Figure 3

**Composition of Mathematics Test by
Topic, Subtopic, and Number of Items**

Topic	Subtopic	Number of Items
Number, Operations, Assumptions	Addition	4
	Subtraction	4
	Multiplication	4
	Division	4
	Laws and Generalizations	4
		<u>20 Total</u>
Geometry	Definitions of terms	4
	Recognition of geometric figures	4
	Measurement of geometric figures	4
		<u>12 Total</u>
Relations	Equality	4
	Order relations	4
	Maps, graphs, charts	4
	Functions	4
		<u>16 Total</u>
Numeration	Sets	4
	Other number bases	4
	Reading, writing, using numbers	4
	Roman numerals	4
		<u>16 Total</u>
Measurement	Concepts	4
	Conversion of units	4
	Operations -- no conversion	4
	Operations -- with conversion	4
		<u>16 Total</u>
Fundamental Operations with Fractions	Concepts	4
	Addition	4
	Subtraction	4
	Multiplication	4
	Division	4
		<u>20 Total</u>

selection of instructional objectives from the total number of instructional objectives presented in the curriculum guide.

3. Reviewing appropriate standardized mathematics tests.
4. Choosing test items to reflect the instructional objectives selected for study. For instructional objectives which were not represented by items on standardized tests, constructing test items.
5. Presenting the selected instructional objectives and the test items to judges¹ for evaluation of the items' appropriateness, relevance, etc.
6. Making appropriate changes in the test items on the basis of recommendations by the judges.

Appendix B consists of the following materials germane to these several steps: the instructional objectives selected for study, designations of the grade levels at which these instructional objectives apply, and the test items chosen to reflect the selected instructional objectives. These materials are organized by the topics and subtopics chosen for study.

Specifically, the mathematics test was composed of 100 items and required 110 minutes to administer. Forty of the 100 items consisted of selected items in Parts II and IV of the School and College Ability Tests, Form 5A (Educational Testing Service, 1957). Subjects were administered the published form of this test which contains a total of 50 items² in Parts II and IV. The remaining 60 test items needed were obtained from the following sources: Sequential Tests of Educational Progress, Forms 3A, 4B (Educational Testing Service, 1957), Contemporary Mathematics Tests: Upper and Lower Elementary Levels, Form A (California Test Bureau, 1963), and 18 items constructed especially for the present study. These 60 items were reproduced in mimeographed form and administered as a supplementary test. This supplementary test is in Appendix C. The source and identification of each of the 100 items selected for inclusion are presented in Appendix D.

¹These judges respectively were specialists in mathematics, tests and measurements, and educational psychology.

²Only 40 of the 50 items were included in the data analyses. Ten items were not relevant to the instructional objective chosen for study.

Reliability of the Mathematics Topic and Subtopic Scores

The reliability coefficients for the mathematics topics and subtopics scores were calculated by the split-half method and corrected by the Spearman-Brown Prophecy Formula. The data were the scores of the total sample at each grade level. The obtained reliability coefficients are presented in Table 1. Inspection of this table reveals that, in general, the reliabilities of the topic scores are sufficiently high to be considered satisfactory for research purposes with groups of subjects (Thorndike, 1951). However, it should be noted that the reliabilities for several of the subtopic scores are somewhat less than satisfactory. In those instances where subtopic reliabilities were too low to be meaningful for the purposes of this study, these subtopics were eliminated from the analyses of the subtopic scores. These subtopic scores, however, were included in the scores for the corresponding total topic and thus were involved in the estimation of the total topic reliability and the analyses related to the research objectives.

The lower reliabilities of the subtopics generally was considered to be primarily a function of the restricted number of items within the subtopics; *i.e.*, there were only four items in each subtopic. This restriction was dictated, to a large extent, by the practical exigencies of the research situation in the schools where a limited amount of testing time was available. While the majority of the subtopic scores were characterized by relatively low reliabilities, they are considered sufficiently high to yield some preliminary information about trends. However, in order to obtain less equivocal results, the study would need to be replicated using subtopic tests of greater length and higher reliability.

Instruments for Assessing Intellectual Processes

Definitions of Terms

The intellectual processes were assessed by instruments designed to involve use of certain intellectual processes in dealing with a variety of materials. Listed below are specifications of terms relevant to the intellectual processes and materials.

Table 1

Reliability Coefficients for the Mathematics Topic and Subtopic Scores^a

Variable	Number of Items	Grade Levels ^b			
		Grade 4 N = 249	Grade 5 N = 267	Grade 6 N = 259	
Number, Operations, and Assumptions	Y1 Addition	4	.459	.276	.387
	Y2 Subtraction	4	.466	.445	.493
	Y3 Multiplication	4	.591	.477	.622
	Y4 Division	4	.394	.467	.465
	Y5 Laws and Generalizations	4	.605	.510	.244
	Y6 Total Topic	20	.810	.696	.751
Geometry	Y7 Definition of Geometric Terms	4	.150	.088	.097
	Y8 Recognition of Geometric Figures	4	.429	.465	.708
	Y9 Measurement of Geometric Figures	4	---	.633	.622
	Y10 Total Topic	12	.421	.683	.711
Relations	Y11 Equality	4	.539	.503	.354
	Y12 Order Relationships	4	.294	.414	.558
	Y13 Maps, Graphs, and Charts	4	.315	.336	.206
	Y14 Functions	4	.140	.467	.562

Table 1 (Continued)

	Variable	Number of Items	Grade Levels ^b		
			Grade 4 N = 249	Grade 5 N = 267	Grade 6 N = 259
Numeration	Y15 Total Topic	16	.593	.732	.764
	Y16 Sets	4	.277	.182	.153
	Y17 Other Number Bases	4	.587	.695	.677
	Y18 Reading, Writing, Using Numbers	4	.326	.480	.569
	Y19 Roman Numerals	4	.540	.608	.644
	Y20 Total Topic	16	.724	.744	.765
Measurement	Y21 Concepts	4	.410	.426	.488
	Y22 Conversion of Units	4	.391	.474	.612
	Y23 Operations -- No Conversion	4	.672	.704	.679
	Y24 Operations -- With Conversion	4	.331	.452	.541
	Y25 Total Topic	16	.765	.783	.830
Fractions	Y26 Concepts	4	.473	.410	.581
	Y27 Addition	4	.060	.533	.549
	Y28 Subtraction	4	.328	.413	.503
	Y29 Multiplication	4	---	---	.765

Table 1 (Continued)

	Variable	Number of Items	Grade Levels ^b		
			Grade 4 N = 249	Grade 5 N = 267	Grade 6 N = 259
Fractions (Continued)	Y30 Division	4	---	---	.299
	Y31 Total Topic	20	.589	.745	.700

^aThe subtopics Measurement of geometric figures, Multiplication of fractions, and Division of fractions in Grade 4, and the subtopics Multiplication of fractions and Division of fractions at Grade 5 are omitted here because they were not taught at these grade levels, respectively. The scores for these subtopics were not summed for the corresponding total topic scores.

^bThis table of reliability coefficients reflects a sample size at each grade level which is different from the size of the samples used in the analyses of the intellectual processes tests and in the regression analyses. This occurs because the mathematics and intellectual processes tests were administered on different days and attrition occurred because of absences. For the regression analyses, only the scores of subjects who were administered the entire battery of tests were analyzed.

- a. Intellectual processes. Activities or operations in the cognitive domain, for example: perceptual discrimination, immediate memory, paired associate memory.
- b. Materials. The content to which the cognitive operations are applied, for example: pictures of concrete objects; drawings and combinations of letters which have no conventional referents; spatial forms and diagrams; numerals and number-quantities; letters and words.

Description of the Tests

Generally, the selection of the intellectual processes to be studied was predicated upon joint consideration of two types of information: (1) explicit descriptions of behavior stated in the instructional objectives for the mathematics topics and subtopics selected for study; and (2) detailed descriptions of the intellectual processes. Designating the intellectual processes for study and selecting the instruments to assess these processes involved the following steps.

1. Detailed review of the instructional objectives for the selected mathematics topics and subtopics.
2. Review of literature pertinent to relationships among mathematics achievement and various intellectual processes.
3. Selection of intellectual processes for study by applying such criteria as relevance, predominance, precision of behavioral specification, etc.
4. Review of appropriate instruments.
5. Choice of tests to sample the intellectual processes selected for study; for tests which required modifications, making the necessary adaptations.

Specifically, the tests used to assess the selected intellectual processes were the following: (1) selected subtests of the Primary Mental Abilities Test for Grades 4-6 (Thurstone and Thurstone, 1962) and (2) adaptations of intellectual processes tests described in various other publications. Figure 4 contains a list of the names of the intellectual processes tests used and the process each test was designed to measure.

Figure 4
Names and Processes Measured by the
Intellectual Processes Tests

Name of Test	Process Measured
Number Comparison	Perceptual Discrimination--Sets of Digits
Number Span	Memory Span
Word Number	Associative Memory
Digit Symbol Equations	Associative Memory plus Conversion
Number Series	Reasoning -- Number Relations
PMA Word Test	Verbal Meaning -- Synonyms
PMA Word Grouping	Reasoning -- Word Grouping
PMA Spatial Relations	Spatial Relations -- Visualization

The subtests selected from the Primary Mental Abilities Test for Grades 4-6 were administered to the subjects in the published form. The adaptations of published intellectual processes tests were reproduced in mimeographed form and administered to the subjects as a supplementary test battery. A copy of each test in the supplementary battery is presented in Appendix E. Presented below is a brief description of each test which was administered. This description is organized in terms of the following: the name of the test, the name of the intellectual process the test was designed to measure, a description of the intellectual process, the source of the test, modifications which were made in the test, and a sample test item.

I. Supplementary Intellectual Processes Tests.

A. Number Comparison: Perceptual discrimination -- sets of digits.

This test assesses speed in finding figures, making comparisons, and carrying out other very simple tasks involving visual perception. It is a modification of the Number Comparison Test (French, Ekstrom, and Price, (1963) which was adapted from the Minnesota Vocational Test for Clerical Workers (Andrew, Paterson, Longstaff, 1933). The following modification was made. The length of each multi-digit number was reduced by 3. Appropriate revisions were made in the directions.

Sample item:

Put an X on the line between the numbers that are not the same.

639	_____	639
471430	_____	471530
6538	_____	6538
710	_____	710

Look at the two groups of numbers with the line between them. Your job is to compare the numbers in these two groups. When the two groups of numbers are not the same, put an X on the line between them. When they are alike, put no mark on the line.

B. Number Span: Memory span.

This test assesses the ability to recall perfectly for immediate reproduction a list or series of items after only one presentation of the series. It is the adaptation of the Number Span Test developed by Kelley (1954) and a modification of the Number Span Test presented by French, Ekstrom, and Price (1963). The following modifications were made. The number of lists presented to subjects was reduced from 20 to 12. The range of the digits presented was reduced from 4--13 to 4--9. This latter modification was based on the lengths of the lists of digits used in the Terman-Merrill revision of the Stanford-Binet Intelligence Scale (Terman and Merrill, 1960). Two different lists were used at each length 4 through 9. Appropriate revisions were made in the directions.

Sample item:

List A	(Administrator - orally)	9	7	2	5
List B	(Administrator - orally)	6	4	1	8

In this test you will work with series of numbers. The numbers will be read aloud to you. After a series of numbers is read to you, write down all the numbers in exactly the same order in which they are read. Do not write any number until the whole list has been read.

C. Word Number -- Associative memory.

This test assesses the ability to form and remember arbitrary associations between discrete stimuli and responses (Underwood, 1949). It is an adaptation of Thurstone's (1938) Word-Number Test. The following modifications were made. The number of word-number pairs was reduced from twenty to eight, and appropriate revisions were made in the directions.

Sample item:

<u>Study List</u>		<u>Test List</u>	
chain	<u>81</u>	shoe	<u> </u>
carrot	<u>72</u>	chain	<u> </u>
shoe	<u>37</u>	map	<u> </u>
map	<u>49</u>	carrot	<u> </u>

Look at the study list of word-number pairs. These are pairs of words and numbers which go together to make a pair; chain goes with 81, carrot goes with 72, and so on. Your task is to learn the words and numbers which go together to make a pair. You will have time to study the list and then the words will be called out to you. Your job will be to write the number which goes with that word to make a pair.

D. Digit Symbol Equations: Associative memory plus conversion.

This test assesses the ability to form and remember arbitrary associations between discrete stimuli and responses and to solve problems involving these associations. It is an adaptation of Ruch's (1934) Nonsense Equation Test. The following modification was made. Symbols other than letters were used. The task was to solve equations which required the substitution of symbols for digits. Appropriate revisions were made in the directions.

Sample item:

In this test you will use symbols and numbers to solve problems.

1	2	3	4	5	6	7	8	9
<	=	X	)	+	#	-	>	(

X and + _____ = and) _____ < and < _____

There are numbers in the top row of boxes; below each number there is a symbol. Each of these number-symbol pairs goes together. Each symbol stands for a number. Below the double line, there are rows of equations. Here, these are combinations of symbols with the answers missing. Your job is to write in the blank space the missing answer to each equation. When you are working the problems, you may look at the top of

the page as often as you like to see which number goes with each symbol.

E. Number Series: Reasoning -- number relations.

This test assesses the associated abilities involved in finding the general rules that will fit sets of data -- the forming and trying out of hypotheses about relationships among numbers. It is an adaptation of the Inductive Reasoning Test from the United States Army Alpha Test (Psychology Committee of the National Research Council, 1917). The following modifications were made. The number of items was increased from 20 to 25; the digits used in each series were changed, but the principle involved remained the same in most instances. Appropriate revisions were made in the directions.

Sample item:

In this test you will be working with series of numbers which have been arranged according to a rule.

1.	6	7	8	9	10	11	_____	_____
2.	35	34	33	32	31	30	_____	_____

Look at the series of numbers in the first row of the sample. Six numbers in the series have been given to you; the blank spaces mean that two numbers in the series are missing. The numbers you see are related in a certain way according to a rule. Your job is to figure out the rule which shows how these numbers are related. Then, you are to use the rule to find the next two numbers in the series, the missing numbers. Write the missing numbers in the blank spaces.

II. Primary Mental Abilities Test for Grades 4-6 (Thurstone and Thurstone, 1962 pp. 4-26). The following descriptions of the abilities measured by the subtests and the examples are excerpted from the PMA examiner's manual.

A. PMA Word Test: Verbal meaning -- synonyms.

This test assesses the ability to understand ideas when they are expressed in words; it is a verbal vocabulary test.

Sample item:

The first word in the following line is SHUT.

SHUT	A. wonder	B. close	C. wipe	D. trade
------	-----------	----------	---------	----------

One of the words means the same as SHUT. This word is Close. Close is answer B. Make a heavy black mark on your answer sheet below the letter B.

B. PMA Word Grouping Test: Reasoning -- word grouping.

This test assesses the ability to identify concepts: to group words on the basis of common attributes.

Sample item:

In the line below three of the words are alike, but one is different. Find the word that is different.

A. Red B. Blue C. Heavy D. Green

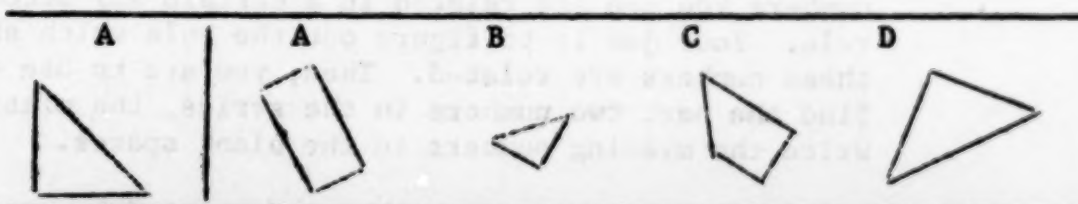
The word that is different is Heavy, since red, blue, and green are all names of colors. Make a black mark in the space below C, because C stands for Heavy, the word that is different.

C. PMA Spatial Relations Test: Spatial relations -- visualization.

This test assesses the ability to visualize objects and figures rotated in space and the relation between them.

Sample item:

In the sample problem below, the first drawing is part of a square.



Look at the other drawings in the row and find the shape that is the other part of the square. Notice that the letter D is above the other part of the square. Make a heavy black mark on your answer sheet in the space below D.

Reliability of the Intellectual Processes Tests

The reliability coefficients for the supplementary intellectual processes tests were calculated by the split-half method and corrected by the Spearman-Brown Prophecy Formula. The data were the scores of the total sample at each grade level. The reliability coefficients for the tests are presented in Table 2. An examination of this table reveals that generally these reliabilities were sufficiently high to be considered

Table 2
Reliability Coefficients for the Intellectual
Processes Tests: Supplementary Battery^a

Name of Test	N	Grade 4	N	Grade 5	N	Grade 6
X4 Equations	265	.817	265	.800	253	.900
X1 Number Comparison	265	.699	270	.789	253	.674
X2 Number Span	265	.634	269	.752	254	.741
X3 Word Number	265	.774	270	.837	254	.843
X5 Number Series	265	.926	270	.948	254	.925

^aThis table of reliability coefficients reflects a sample size at each grade level which is different from the size of the samples used in the regression analyses. This occurs because the intellectual processes tests and the mathematics tests were administered on different days. For the regression analyses, only the scores of subjects who were administered the entire battery of tests were used.

satisfactory for research purposes (Thorndike, 1951).

Excerpted in Table 3 are reliability data for the Primary Mental Abilities Test for Grades 4-6 (Thurstone and Thurstone, 1962) which were presented in the publisher's Technical Report (Science Research Associates, 1965). These reliability data were obtained by the test - retest method. A reliability coefficient for the PMA Spatial Relations Test was presented in the publisher's Technical Report. This was not the case for the PMA Words Test and the PMA Word Grouping Test. Instead, reliability coefficients were presented for the verbal meaning and reasoning abilities; each of these reliability coefficients was based, presumably, on combined scores for two subtests. The data on the verbal meaning and reasoning abilities are presented herein to portray a general notion of the kind of reliability coefficients obtained. Reliability coefficients for the PMA Words Test and the PMA Word Grouping Test probably would not be as high as those portrayed in Table 3. However, since all of the reliability coefficients which were presented for the PMA tests were generally high, and since the PMA Words Test consisted of 30 items and the PMA Word Grouping Test consisted of 25 items, it seems reasonable to assume that the reliability coefficients for these subtests were sufficiently high for research purposes with groups of subjects.

General Procedures

Major steps involved in data collection included the following activities. Classes of pupils were selected and eligible subjects were identified. Prior to the administration of the tests, a detailed testing schedule was formulated with the cooperation of appropriate school personnel. The testing sessions were scheduled so that all selected classes were administered the tests at approximately the same time. Meetings were held with teachers and principals in order to orient them to the project and to provide them with materials and information regarding test schedules and testing procedures. Appendix F contains a copy of the Project Information Guide for Teachers; this guide was given to teachers at the first meeting. Each teacher also was provided with an administrator's notebook which contained complete sets of directions and test booklets for all tests. Following the orientation sessions, conferences were arranged for individual teachers who wished additional

Table 3

**Reliability Coefficients for the Primary
Mental Abilities Tests**

Subtest	Grade 4		Grade 5		Grade 6	
	Retest Interval		Retest Interval		Retest Interval	
	1 Week	4 Weeks	1 Week	4 Weeks	1 Week	4 Weeks
	N = 33	N = 33	N = 31	N = 28	N = 28	N = 27
	r_{ii}	r_{ii}	r_{ii}	r_{ii}	r_{ii}	r_{ii}
X6 Verbal Meaning (Words and Pictures)	.92	.96	.95	.91	.91	.93
X7 Spatial Relations	.75	.65	.87	.72	.68	.67
X8 Reasoning (Words and Figures)	.83	.73	.89	.88	.77	.77

assistance in preparing for the test administration.

Test materials for pupils were prepared, packaged, and delivered to the schools prior to each test administration session. The instruments selected to sample the intellectual processes and mathematics achievement were administered to all pupils in the specified classes. All testing materials were collected from the respective schools immediately following each test administration session. Tests taken by pupils identified as not having met the selection criteria were eliminated.

Analyses

The data consisted of interval scores (Cronbach, 1960; Underwood, 1957) obtained on the mathematics achievement tests (Y-variables) and the intellectual processes tests (X-variables).

In addition to the analyses immediately germane to the four research objectives, analyses were made of the fourth, fifth, and sixth grade pupils' relative performance on the mathematics topics and subtopics and on the intellectual processes tests. The purpose of these analyses was to obtain background data pertinent to the analyses specific to the four research objectives. The performance of each group was described in terms of means and standard deviations. A single classification analysis of variance was used to analyze relationships among the groups' means (Steele and Torrie, 1960). When the *F*-ratios were significant the following techniques were used for making comparisons: *t*-tests of differences between means and *F*-tests of homogeneity of the within-groups variances (Snedecor, 1956; McNemar, 1955).

The .05 probability level was the criterion for rejecting the null hypothesis related to means; the .001 probability level was the criterion for rejecting the null hypothesis related to within-group variances. When the null hypothesis pertaining to within-group variances was rejected, extreme heterogeneity was inferred to be present between the variances of the groups being compared; in these cases, when the relationship between the involved groups' means was being evaluated, the criterion for rejecting the null hypothesis was changed to the .01 probability level (Cochran and Cox, 1957; Lindquist, 1953; Snedecor, 1956).

Step-wise regression analyses were considered appropriate for the generalized treatment of the data specific to the four research objectives (Cooley and Lohnes, 1962; Efroymson, 1960; Steele and Torrie, 1960). The criterion for rejecting the null hypothesis in all cases was the .05 probability level. The analyses were performed separately for each mathematics subtopic and topic at each grade level. IBM computers were used for these analyses.

When the regression equations were specified by the step-wise regression analyses, the percentage contribution of each X-variable in the equation to the Y-variable was determined in the following way: the zero-order r 's for the particular Y-variable and the X-variable involved was multiplied by the corresponding standard partial regression coefficient (beta weight). Thus the percentage contributions presented in Chapter III contain not only the separate coefficients of determination, but also the shared coefficients of determination. This procedure was the same as that employed by Holmes and Singer (1961, p.87). Their comments concerning the use of this technique are pertinent here.

...it will be realized that each of the preferential predictors selected because it made an independent contribution to the variance of the criterion also entered into a joint relationship with other selected predictors to the end that each shared in what can be thought of as an additional contribution to the criterion's variance. This shared variance can be divided between the partners by prorating the total amount in terms of the coefficient of separate determination attributable to each, or by simply dividing the shared amount equally and assigning each half to the appropriate partners. The technique employed in the present study of multiplying the betas by the zero-order r 's is tantamount to the latter alternative; that is, using the coefficient of separate determination plus one-half of the coefficient of shared determination in Wright's (1921) sense in order to account for the total influence (independent and shared) that a preferential predictor might exert on the criterion. This is justified according to Ezekial (1953), since these two methods have a mathematical equality. (Italics are Holmes' and Singer's.)

Because of problems related either to a subtopic not being taught at a certain grade level or to low reliability, analyses of several mathematics scores were not warranted. These problems are elaborated on below.

The first problem was concerned with the fact that some of the subtopics on the mathematics test had not been taught at certain grade levels. Since the mathematics test was administered to all three grade

levels and all subjects responded to all of the items, the question here was how to treat the scores for those subtopics which had not been taught. It was decided to exclude from the analyses of the separate subtopics the scores for those subtopics which had not been taught. In addition, these subtopics scores were not included in the scores for their respective total topic analyses. The following subtopics were involved: Fourth grade -- Measurement of geometric figures, Multiplication of fractions, and Division of fractions; Fifth grade -- Multiplication of fractions and Division of fractions. The sixth grade subjects had been taught all subtopics.

The second problem was related to unsatisfactory reliability for some of the mathematics subtopics. It was noted in the section, Reliability of the Mathematics Topic and Subtopic Scores, that several of the subtopic reliabilities were unsatisfactory for research purposes. These were the following: Fourth grade -- Definitions of geometric terms, Functions, and Addition of fractions; Fifth grade -- Definitions of geometric terms; Sixth grade -- Definitions of geometric terms. Therefore, these subtopic scores were eliminated from the analyses of the separate subtopics. However, these subtopic scores were included in the scores for their respective total topic analyses.

CHAPTER III

RESULTS

Introduction

The present chapter is concerned with statistical data relevant to the purpose of this investigation: namely, to examine relationships between certain intellectual processes and achievement of instructional objectives for selected mathematics topics and subtopics taught at grade levels four, five, and six. The presentation of the statistical data is organized as follows. The detailed basic statistical source data are in the appendices: Appendix G contains the simple or zero-order correlations between the fourth, fifth, and sixth grade subjects' scores on the mathematics and intellectual processes tests; Appendix H consists of results of the step-wise regression analyses. The statistical data¹ cited within the present chapter are grouped into five sections. The first section consists of descriptive and inferential statistics reflecting the central tendency and variability of each group's performance on the mathematics and intellectual processes tests. The remaining four sections include data pertaining to the specific research objectives which were guides to the collection of evidence germane to the purpose. The specific research objectives were the following.

1. To identify intellectual processes which are related, singly or in combination, to achievement of instructional objectives for each mathematics topic and subtopic.
2. To determine to what extent the identified intellectual processes or combinations of intellectual processes are related to achievement of the instructional objectives for each mathematics topic and subtopic.
3. To specify the relative contribution, or weighting, of each intellectual process when combinations of intellectual processes are related to achievement of instructional objectives for each mathematics topic and subtopic.

¹Whenever possible, only prime data are reported. Additional statistics may be obtained by appropriate algebraic and arithmetic operations.

4. To determine whether intellectual processes related to achievement differ among mathematics topics and subtopics.

The data for the four research objectives are closely related. Essentially, these data are pertinent statistics excerpted from the results of the step-wise regression analyses (Appendix H) and results of further treatment of these statistics according to procedures referred to in the Analyses section of Chapter II. For the first three objectives, the data are presented only in tabular form; no attempt is made to recount verbally the detailed evidence which appears in the tables. Then, for the fourth objective, implications of data pertinent to the first three objectives are summarized both in tabular form and in the text. In Chapter IV, the results are summarized in more general form and discussed.

Descriptive and Inferential Statistics Relevant to the Groups' Performance on the Tests

Mathematics Topics and Subtopics

Means and standard deviations were analyzed for the purpose of determining the fourth, fifth, and sixth grade groups' relative achievement on the mathematics test. Pertinent data are presented in Tables 4, 5, and 6. Table 4 includes means and standard deviations for the topic and subtopic scores; Table 5 contains results of analyses of variance among means for the several topic and subtopic scores; Table 6 consists of results of paired comparisons between the means and the variances of the topic and subtopic scores.

In the section on Analyses in Chapter II, certain mathematics scores were specified for deletion from statistical treatment because of problems related either to a subtopic's score having insufficiently high reliability or to the subtopic's not being taught. Implications of these deletions for analyses of mean performance at each grade level are listed below according to the topics and subtopics which were affected.

1. Definitions of geometric terms (Y7). The reliability coefficients for this subtopic's scores at all grade levels were too low to warrant analysis.

Table 4

Means and Standard Deviations: Fourth, Fifth, and Sixth Grade Groups' Performance on the Mathematics Test^a

	Variable	Grade 4 n = 244		Grade 5 n = 261		Grade 6 n = 240	
		$\bar{X}$	S	$\bar{X}$	S	$\bar{X}$	S
Number, Operations, Assumptions	Y1 Addition	3.16	.96	3.24	.88	3.27	.89
	Y2 Subtraction	3.11	1.00	3.25	.92	3.27	.83
	Y3 Multiplication	1.87	1.22	2.59	1.19	3.19	1.04
	Y4 Division	1.41	.94	2.02	1.13	2.70	1.12
	Y5 Laws and Generalizations	2.94	.96	2.67	.96	2.91	.80
	Y6 Total Topic	12.50	3.34	13.75	3.39	15.54	3.04
Geometry	Y7 Definition of Terms	--	--	--	--	--	--
	Y8 Recognition of Geometric Figures	2.99	1.04	2.94	1.05	3.26	.96
	Y9 Measurement of Geometric Figures	--	--	1.95	1.14	2.80	1.17
	Y10 Total Topic	4.73	1.56	6.79	2.36	7.96	2.39
Relations	Y11 Equality	3.17	.96	3.02	1.06	3.23	.91
	Y12 Order Relations	1.12	1.02	1.68	1.14	2.27	1.27
	Y13 Maps, Graphs, Charts	2.45	1.06	2.46	1.02	2.78	.96

Table 4 (Continued)

	Variable	Grade 4 n = 244		Grade 5 n = 261		Grade 6 n = 240	
		$\bar{X}$	S	$\bar{X}$	S	$\bar{X}$	S
Relations (Continued)	Y14 Functions	--	--	2.28	1.17	2.67	1.18
	Y15 Total Topic	8.68	2.75	9.43	3.06	10.94	3.12
Numeration	Y16 Sets	2.64	1.07	2.13	1.09	2.21	1.03
	Y17 Number Bases	1.46	1.29	1.89	1.37	1.52	1.33
	Y18 Reading, Writing, Using Numbers	2.26	1.09	2.49	1.10	2.85	1.12
	Y19 Roman Numerals	1.98	1.18	2.31	1.30	2.62	1.21
	Y20 Total Topic	8.33	3.32	8.83	3.48	9.19	3.15
Measurement	Y21 Concepts of Measurement	1.70	1.17	2.03	1.20	2.40	1.20
	Y22 Conversion of Units	2.04	1.03	2.22	1.10	2.73	1.10
	Y23 Operations -- No Conversion	3.00	1.04	3.38	1.00	3.61	.77
	Y24 Operations -- With Conversion	1.62	1.10	1.89	1.17	2.35	1.10
	Y25 Total Topic	8.37	3.15	9.52	3.37	11.09	3.31

Table 4 (Continued)

	Variable	Grade 4 n = 244		Grade 5 n = 261		Grade 6 n = 240	
		$\bar{X}$	S	$\bar{X}$	S	$\bar{X}$	S
Fundamental Operations with Fractions	Y26 Concepts	1.55	1.05	1.93	1.04	3.13	.99
	Y27 Addition	--	--	2.60	1.03	3.16	.97
	Y28 Subtraction	1.58	1.91	2.39	1.12	3.06	1.04
	Y29 Multiplication	--	--	--	--	2.78	1.11
	Y30 Division	--	--	--	--	2.06	1.08
	Y31 Total Topic	4.95	2.22	6.93	2.53	14.14	3.90

^aSee the text immediately above and section on Analyses for explanation of deleted statistics.

Table 5

Results of Analyses of Variance Among Means: Fourth, Fifth, and Sixth Grade Groups' Performance on the Mathematics Test^a

Variable	Source of Variation	Degrees of Freedom	Mean Square
Y1 Addition	Between Within	2 742	.68 ^{ns} .83
Y2 Subtraction	Between Within	2 742	7.82*** .85
Y3 Multiplication	Between Within	2 742	105.54*** 1.33
Y4 Division	Between Within	2 742	101.39*** 1.15
Y5 Laws and Generalizations	Between Within	2 742	5.76** .85
Y6 Total Topic: Number, Operations, Assumptions	Between Within	2 742	563.52*** 10.64
Y7 Definitions of Terms	Between Within	-- --	-- --
Y8 Definition of Geometric Figures	Between Within	2 742	7.16** 1.03
Y9 Measurement of Geometric Figures	Between Within	2 742	102.46*** 1.18
Y10 Total Topic: Geometry	Between Within	-- --	-- --
Y11 Equality	Between Within	2 742	3.10* .94
Y12 Order Relations	Between Within	2 742	79.73*** 1.32
Y13 Maps, Graphs, Charts	Between Within	2 742	8.31*** 1.02
Y14 Functions	Between Within	-- --	-- --

Table 5 (Continued)

Variable	Source of Variation	Degrees of Freedom	Mean Square
Y15 Total Topic: Relations	Between Within	2 742	320.13*** 8.90
Y16 Sets	Between Within	2 742	18.32*** 1.14
Y17 Number Bases	Between Within	2 742	14.12*** 1.77
Y18 Reading, Writing, Using Numbers	Between Within	2 742	21.20*** 1.21
Y19 Roman Numerals	Between Within	2 742	24.59*** 1.52
Y20 Total Topic: Numeration	Between Within	2 742	45.17*** 11.06
Y21 Concepts of Measurement	Between Within	2 742	29.25*** 1.42
Y22 Conversion of Units	Between Within	2 742	31.62*** 1.16
Y23 Operations - No Conversion	Between Within	2 742	22.57*** .90
Y24 Operations - With Conversion	Between Within	2 742	32.68*** 1.27
Y25 Total Topic: Measurement	Between Within	2 742	451.69*** 10.76
Y26 Concepts	Between Within	2 742	164.14*** 1.05
Y27 Addition	Between Within	-- --	-- --
Y28 Subtraction	Between Within	2 742	124.71*** 1.13
Y29 Multiplication	Between Within	-- --	-- --

Table 5 (Continued)

Variable	Source of Variation	Degrees of Freedom	Mean Square
Y30 Division	Between Within	-- --	-- --
Y31 Total Topic: Fractions	Between Within	-- --	-- --

*P < .05; **P < .01; ***P < .001

^aSee the text immediately above and the section on Analyses for explanation of deleted statistics.

Table 6
Results of Paired Comparisons Between Means and Variances: Fourth, Fifth, and Sixth
Grade Groups' Performance on the Mathematics Test^{a,b,c}

Variable	Comparison	$F(S^2/S^2)$	$S^2_X - \bar{X}$	$\bar{X} - \bar{X}$	Relationship
Y1 Addition	G4:G5	1.18	--	--	G5=G4
	G4:G6	1.15	--	--	G6=G4
	G5:G6	1.02	--	--	G6=G5
Y2 Subtraction	G4:G5	1.20	.06	.13*	G5>G4
	G4:G6	1.47	.06	.36***	G6>G4
	G5:G6	1.23	.06	.22***	G6>G5
Y3 Multiplication	G4:G5	1.06	.08	.72***	G5>G4
	G4:G6	1.38	.07	1.32***	G6>G4
	G5:G6	1.31	.07	.60***	G6>G5
Y4 Division	G4:G5	1.44	.06	.62***	G5>G4
	G4:G6	1.42	.07	1.29***	G6>G4
	G5:G6	1.02	.07	.68***	G6>G5
Y5 Laws & Generalizations	G4:G5	1.00	.06	.28***	G5<G4
	G4:G6	1.44	.06	.03	G6=G4
	G5:G6	1.45	.06	.24***	G6>G5
Y6 Total Topic Number, Operations, Assumptions	G4:G5	1.03	.21	1.25***	G5>G4
	G4:G6	1.21	.21	3.04***	G6>G4
	G5:G6	1.25	.20	1.78***	G6>G5

Table 6 (Continued)

Variable	Comparison	$F(S^2/S^2)$	$S^2 - \bar{X}$	$\bar{X} - \bar{X}$	Relationship
Y7 Definitions of Terms	G4:G5 G4:G6 G5:G6	-- -- --	-- -- --	-- -- --	-- -- --
Y8 Recognition of Geometric Figures	G4:G5 G4:G6 G5:G6	1.03 1.17 1.20	.07 .06 .06	.05 .27*** .32***	G5=G4 G6>G4 G6>G5
Y9 Measurement of Geometric Figures	G4:G5 G4:G6 G5:G6	-- -- 1.07	-- -- .07	-- -- .85***	-- -- G6>G5
Y10 Total Topic: Geometry	G4:G5 G4:G6 G5:G6	-- -- --	-- -- --	-- -- --	-- -- --
Y11 Equality	G4:G5 G4:G6 G5:G6	1.15 1.11 1.28	.06 .06 .06	.16* .06 .21***	G5<G4 G6=G4 G6>G5
Y12 Order Relations	G4:G5 G4:G6 G5:G6	1.26 1.56 1.24	.07 .07 .08	.56*** 1.15*** .59***	G5>G4 G6>G4 G6>G5
Y13 Maps, Graphs, Charts	G4:G5 G4:G6 G5:G6	1.08 1.21 1.12	.07 .07 .06	.01 .32*** .32***	G5=G4 G6>G4 G6>G5

Table 6 (Continued)

Variable	Comparison	$F(S^2/S^2)$	$S\bar{X} - \bar{X}$	$\bar{X} - \bar{X}$	Relationship
Y14					
Functions	G4:G5	--	--	--	--
	G4:G6	--	--	--	--
	G5:G6	1.03	.07	.39***	G6>G5
Y15					
Total Topic:	G4:G5	1.24	.18	.74***	G5>G4
Relations	G4:G6	1.28	.19	2.26***	G6>G4
	G5:G6	1.04	.20	1.51***	G6>G5
Y16					
Sets	G4:G5	1.05	.07	.50***	G5<G4
	G4:G6	1.06	.07	.43***	G6<G4
	G5:G6	1.12	.07	.08	G6=G5
Y17					
Number Bases	G4:G5	1.13	.08	.43***	G5>G4
	G4:G6	1.06	.08	.06	G6=G4
	G5:G6	1.08	.09	.38***	G6<G5
Y18					
Reading, Writing, Using Numbers	G4:G5	1.01	.07	.23**	G5>G4
	G4:G6	1.05	.07	.59***	G6>G4
	G5:G6	1.04	.07	.36***	G6>G5
Y19					
Roman Numerals	G4:G5	1.20	.08	.33***	G5>G4
	G4:G6	1.05	.08	.63***	G6>G4
	G5:G6	1.15	.08	.30***	G6>G5
Y20					
Total Topic: Numeration	G4:G5	1.10	.21	.50*	G5>G4
	G4:G6	1.12	.21	.86***	G6>G4
	G5:G6	1.23	.21	.36	G6=G5

Table 6: (Continued)

Variable	Comparison	$F(S^2/S^2)$	$S^2 - \bar{X}$	$\bar{X} - \bar{X}$	Relationship
Y21					
Concepts of Measurement	G4:G5	1.05	.08	.33***	G5>G4
	G4:G6	1.06	.08	.70***	G6>G4
	G5:G6	1.00	.08	.37***	G6>G5
Y22					
Conversion of Units	G4:G5	1.14	.07	.18**	G5>G4
	G4:G6	1.14	.07	.70***	G6>G4
	G5:G6	1.00	.07	.51***	G6>G5
Y23					
Operations no	G4:G5	1.08	.06	.38***	G5>G4
Conversions	G4:G6	1.81*	.06	.60***	G6>G4
	G5:G6	1.68*	.06	.23***	G6>G5
Y24					
Operations with	G4:G5	1.13	.07	.27***	G5>G4
Conversions	G4:G6	1.00	.07	.72***	G6>G4
	G5:G6	1.13	.07	.46***	G6>G5
Y25					
Total Topic: Measurement	G4:G5	1.14	.21	1.16***	G5>G4
	G4:G6	1.10	.21	2.72***	G6>G4
	G5:G6	1.04	.21	1.57***	G6>G5
Y26					
Concepts	G4:G5	1.02	.07	.39***	G5>G4
	G4:G6	1.13	.07	1.58***	G6>G4
	G5:G6	1.11	.06	1.19***	G6>G5
Y27					
Addition	G4:G5	--	--	--	--
	G4:G6	--	--	--	--
	G5:G6	1.13	.06	.56***	G6>G5

Table 6 (Continued)

Variable	Comparison	$F(S^2/S^2)$	$S^2 - \bar{X}$	$\bar{X} - \bar{X}$	Relationship
Y28 Subtraction	G4:G5	1.24	.07	.81***	G5>G4
	G4:G6	1.07	.07	1.43***	G6>G4
	G5:G6	1.16	.07	.62***	G6>G5
Y29 Multiplication	G4:G5	--	--	--	--
	G4:G6	--	--	--	--
	G5:G6	--	--	--	--
Y30 Division	G4:G5	--	--	--	--
	G4:G6	--	--	--	--
	G5:G6	--	--	--	--
Y31 Total Topic: Fractions	G4:G5	--	--	--	--
	G4:G6	--	--	--	--
	G5:G6	--	--	--	--

*P < .05; **P < .01; ***P < .001

^aThe degrees of freedom for the F - tests of homogeneity of variance (MS within) were the following: Grade 4---243; Grade 5---260; Grade 6---239.

^bThe degrees of freedom for the t - tests of differences between means were the following for these comparisons: Grade 4 and Grade 5---503; Grade 4 and Grade 6---482; Grade 5 and Grade 6---499.

^cSee the text immediately above and the section on Analyses for explanation of deleted statistics.

2. Functions (Y14) and Addition of Fractions (Y27). At the fourth grade level, the reliability coefficients for the scores for these subtopics were insufficiently high to warrant analysis. Consequently, t-tests were used to analyze differences between the means at the fifth and sixth grade levels.
3. Measurement of geometric figures (Y9). At the fourth grade level, this subtopic was not taught. And so, t-tests were used to analyze differences between means at the fifth and sixth grade levels.
4. Multiplication of fractions (Y29) and Division of fractions (Y30). At the fourth and fifth grade levels, these subtopics were not taught. Therefore, no comparisons between grade levels were appropriate.
5. Total Topic: Geometry (Y10). Because Measurement of geometric figures was not taught at the fourth grade level, the scores for this subtopic were not included in the total scores. As a result, the total topic scores for Geometry at the fourth grade level was not comparable to the corresponding scores at the fifth and sixth grade levels. Differences between means for the Geometry topic scores at the fifth and sixth grade levels were analyzed by t-tests.
6. Total Topic: Fractions (Y31). Because Multiplication of fractions and Division of fractions were not taught at the fourth and fifth grade levels, the scores for these subtopics did not enter into the total topic scores. Therefore, the total topic scores for Fractions was not comparable at the fourth, fifth, and sixth grade levels. It follows that an analysis of differences among means was not appropriate.

The results of the statistical analyses indicated the following: although some exceptions occurred, the groups generally differed in the expected directions on the majority of mathematics topics and subtopics analyzed. That is, the means for grade six exceeded the means for grade five and the means for grade five and grade six exceeded the means for grade four.

Intellectual Processes Tests

Means and standard deviations were analyzed for the purpose of determining the fourth, fifth, and sixth grade groups' relative performance on the intellectual processes tests. Pertinent data are presented in Tables 7, 8, and 9. Table 7 contains means and standard deviations for the intellectual processes test scores; Table 8 includes results of analyses of variance among means; Table 9 is composed of the results of paired comparisons between the means and the variances of groups' scores on the intellectual processes tests. The data in these tables indicate that, generally, on the majority of the intellectual processes tests, the groups differed in the expected directions; that is, the means for grade six exceeded the means for grade five and the means for grades five and six exceeded the means for grade four.

85.2	82.11	81.0	81.11	80.0	77.01	
85.0	82.75	81.0	80.75	82.0	80.00	
85.2	80.40	80.0	81.00	81.2	81.71	
85.0	80.01	82.0	80.01	81.0	80.01	
86.2	80.00	82.0	81.01	81.0	80.01	

Table 7

Means and Standard Deviations: Fourth, Fifth, and Sixth Grade Groups' Performance on the Intellectual Processes Tests

Variable	Grade 4 n = 244		Grade 5 n = 261		Grade 6 n = 240	
	$\bar{X}$	S	$\bar{X}$	S	$\bar{X}$	S
X4 Associative Memory -- Conversion	15.41	5.82	14.88	5.66	17.83	10.82
X1 Perceptual Discrimination	11.59	4.33	12.78	4.74	13.71	4.10
X2 Memory Span	6.45	1.53	6.57	1.68	7.13	1.53
X3 Associative Memory	10.17	4.44	11.14	4.10	11.26	4.45
X5 Reasoning -- Number Relations	20.85	8.54	23.99	9.79	27.28	8.23
X6 Verbal Meaning -- Synonyms	17.72	6.17	20.18	6.36	24.07	5.29
X7 Spatial Relations	12.76	4.06	13.66	4.35	14.62	4.47
X8 Reasoning -- Word Grouping	17.36	4.08	18.20	4.37	20.03	3.55

Table 8

Results of Analyses of Variance Among Means: Fourth, Fifth, and Sixth Grade Groups' Performance on the Intellectual Processes Tests

Variable	Source of Variation	Degrees of Freedom	Mean Square
X4 Associative Memory -- Conversion	Between Within	2 742	603.09*** 60.05
X1 Perceptual Discrimination	Between Within	2 742	271.92*** 19.43
X2 Memory Span	Between Within	2 742	31.41*** 2.52
X3 Associative Memory	Between Within	2 742	88.22*** 18.73
X5 Reasoning -- Number relations	Between Within	2 742	2,498.82*** 79.35
X6 Verbal Meaning -- Synonyms.	Between Within	2 742	2,476.32*** 35.64
X7 Spatial Relations	Between Within	2 742	208.09*** 18.48
X8 Reasoning -- Word Grouping	Between Within	2 742	451.90*** 16.18

*P < .05; **P < .01; ***P < .001

Table 9
Results of Paired Comparisons Between Means and Variances: Fourth, Fifth, and Sixth
Grade Groups' Performance on the Intellectual Processes Tests^{a, b}

Variable	Comparison	$F(S^2/S^2)$	$S^2 - \bar{X}$	$\bar{X} - \bar{X}$	Relationship
X4 Associative Memory -- Conversion	G4:G5	1.06	.36	.53	G5=G4
	G4:G6	3.46***	.56	2.41***	G6>G4
	G5:G6	3.65***	.55	2.94***	G6>G5
X1 Perceptual Discrimination	G4:G5	1.20	.29	1.18**	G5>G4
	G4:G6	1.11	.27	2.11***	G6>G4
	G5:G6	1.33	.28	.93***	G6>G5
X2 Memory Span	G4:G5	1.20	.10	.12	G5=G4
	G4:G6	1.00	.10	.67***	G6>G4
	G5:G6	1.20	.10	.55***	G6>G5
X3 Associative Memory	G4:G5	1.77*	.27	.97***	G5>G4
	G4:G6	1.01	.29	1.09***	G6>G4
	G5:G6	1.18	.27	.12	G6=G5
X5 Reasoning -- Number Relations	G4:G5	1.31	.58	3.14***	G5>G4
	G4:G6	1.08	.54	6.43***	G6>G4
	G5:G6	1.41	.57	3.29***	G6>G5

Table 9 (Continued)

Variable	Comparison	$F(S^2/S)$	$S^2 - \bar{X}$	$\bar{X} - \bar{X}$	Relationship
X6 Verbal Meaning -- Words	G4:G5	1.06	.39	2.45***	G5>G4
	G4:G6	1.36	.37	6.35***	G6>G4
	G5:G6	1.45	.37	3.89***	G6>G5
X7 Spatial Relations	G4:G5	1.15	.27	.89***	G5>G4
	G4:G6	1.21	.27	1.85***	G6>G4
	G5:G6	1.05	.28	.96***	G6>G5
X8 Reasoning -- Word Grouping	G4:G5	1.15	.27	.84**	G5>G4
	G4:G6	1.32	.25	2.67***	G6>G4
	G5:G6	1.52	.25	1.83***	G6>G5

* $P < .05$; ** $P < .01$; *** $P < .001$

^aThe degrees of freedom for the F - tests of homogeneity of variance (MS within) were the following: Grade 4--243; Grade 5--260; Grade 6--239.

^bThe degrees of freedom for the t - tests of differences between means were the following for these comparisons: Grade 4 and Grade 5--503; Grade 4 and Grade 6--482; Grade 5 and Grade 6--499.

Research Objective 1
Identification of the Intellectual Processes Contributing
to the Mathematics Topics and Subtopics

The first research objective was to identify intellectual processes which are related, singly or in combination, to achievement of instructional objectives for each mathematics topic and subtopic. The data which are most immediately relevant to this objective are the partial regression coefficients and the corresponding t -ratios obtained in the step-wise regression analyses (Appendix H). These data are listed in Tables 10, 11, and 12. Table 10 includes data for the fourth grade level; Table 11, for the fifth grade level; Table 12, for the sixth grade level.

Grade	Topic	Subtopic	Stepwise Regression		Partial Regression Coefficient	t -Ratio
			R^2	F		
Fourth	Addition	Whole Numbers	0.25	1.25	0.15	1.37
		Decimals	0.34	1.34	0.15	1.37
		Fractions	0.12	1.12	0.15	1.37
Fifth	Addition	Whole Numbers	0.02	1.02	0.15	1.37
		Decimals	0.34	1.34	0.15	1.37
		Fractions	0.12	1.12	0.15	1.37
Sixth	Addition	Whole Numbers	0.02	1.02	0.15	1.37
		Decimals	0.34	1.34	0.15	1.37
		Fractions	0.12	1.12	0.15	1.37

Table 10

Partial Regresssion Coefficients and t -ratios: Grade 4^a

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y1 Addition	X1 Perceptual Discrimination -- Sets of Digits	0.035	2.480*
	X3 Associative Memory	0.039	2.877**
Y2 Subtraction	X3 Associative Memory	0.031	2.313*
	X5 Reasoning -- Number Relations	0.033	3.914***
	X6 Verbal Meaning -- Synonyms	0.034	2.990**
Y3 Multiplication	X5 Reasoning -- Number Relations	0.044	4.459***
	X6 Verbal Meaning -- Synonyms	0.048	3.506***
Y4 Division	X5 Reasoning -- Number Relations	0.036	4.999***
	X7 Spatial Relations -- Visualization	0.032	2.098*
Y5 Laws and Generalizations	X5 Reasoning -- Number Relations	0.035	4.512***
	X6 Verbal Meaning -- Synonyms	0.040	3.761***
Y6 Total Topic: Number, Operations, Assumptions	X3 Associative Memory	0.079	1.999*
	X5 Reasoning -- Number Relations	0.159	6.468***
	X6 Verbal Meaning -- Synonyms	0.138	4.128***
Y7 Definition of Terms	The reliability of this subtopic was insufficient to warrant its inclusion in the analyses.		

Table 10 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y8 Recognition of Geometric Figures	X6 Verbal Meaning -- Synonyms	0.059	5.794***
Y9 Measurement of Geometric Figures	Since this subtopic was not taught at this grade level, it was not included in the analyses.		
	X1 Perceptual Discrimination -- Sets of Digits	0.062	2.668**
Y10 Total Topic: Geometry	X2 Memory Span	-0.136	2.143*
	X6 Verbal Meaning -- Synonyms	0.076	4.863***
	X2 Memory Span	0.095	2.842**
Y11 Equality	X5 Reasoning -- Number Relations	0.043	5.917***
	X6 Verbal Meaning -- Synonyms	0.039	3.809***
Y12 Order Relations	X1 Perceptual Discrimination -- Sets of Digits	0.028	1.981*
	X6 Verbal Meaning -- Synonyms	0.053	5.314***
Y13 Maps, Graphs, Charts	X5 Reasoning -- Number Relations	0.022	2.379*
	X6 Verbal Meaning -- Synonyms	0.040	3.125**
Y14 Functions	The reliability of this subtopic was insufficient to warrant its inclusion in the analyses.		
	X3 Associative Memory	0.076	2.349*
Y15 Total Topic: Relations	X5 Reasoning -- Number Relations	0.086	4.267***
	X6 Verbal Meaning -- Synonyms	0.177	6.452***

Table 10 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y16 Sets	X5 Reasoning -- Number Relations	0.059	8.427***
	X4 Associative Memory Plus Conversion	0.035	2.407*
	X1 Perceptual Discrimination -- Sets of Digits	0.037	1.980*
Y17 Number Bases	X6 Verbal Meaning -- Synonyms	0.045	3.013**
	X8 Reasoning -- Word Grouping	0.064	2.795**
Y18 Reading, Writing, Using Numbers	X5 Reasoning -- Number Relations	0.072	10.286***
	X4 Associative Memory Plus Conversion	0.028	2.306*
	X2 Memory Span	0.093	2.107*
Y19 Roman Numerals	X5 Reasoning -- Number Relations	0.036	3.619***
	X6 Verbal Meaning -- Synonyms	0.046	3.502***
	X4 Associative Memory Plus Conversion	0.097	3.267***
Y20 Total Topic: Numeration	X1 Perceptual Discrimination -- Sets of Digits	0.106	2.813**
	X5 Reasoning -- Number Relations	0.099	4.415***
	X6 Verbal Meaning -- Synonyms	0.166	5.214***
	X8 Reasoning -- Word Grouping	0.127	2.698**

Table 10 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y21 Concepts of Measurement	X4 Associative Memory Plus Conversion	0.025	1.972*
	X2 Memory Span	0.111	2.428*
	X5 Reasoning -- Number Relations	0.031	3.024**
	X6 Verbal Meaning -- Synonyms	0.035	2.539*
Y22 Conversion of Units	X5 Reasoning -- Number Relations	0.041	4.959***
	X6 Verbal Meaning -- Synonyms	0.040	3.519***
Y23 Operations - No Conversion of Units	X4 Associative Memory Plus Conversion	0.037	3.799***
	X2 Memory Span	0.081	2.299*
	X5 Reasoning -- Number Relations	0.030	3.598***
	X6 Verbal Meaning -- Synonyms	0.033	2.804**
	X8 Reasoning -- Word Grouping	0.042	2.500**
Y24 Operations - Conversion of Units	X2 Memory Span	0.086	2.072*
	X5 Reasoning -- Number Relations	0.029	3.159**
	X6 Verbal Meaning -- Synonyms	0.053	4.208***
Y25 Total Topic: Measurement	X4 Associative Memory Plus Conversion	0.072	2.644**
	X2 Memory Span	0.264	2.671**
	X5 Reasoning -- Number Relations	0.133	5.949***
	X6 Verbal Meaning -- Synonyms	0.174	5.853***

Table 10 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y26 Concepts of Fractions	X1 Perceptual Discrimination -- Sets of Digits	0.038	2.601**
	X2 Memory Span	0.111	2.724**
	X3 Associative Memory	0.028	2.025*
	X6 Verbal Meaning -- Synonyms	0.047	4.636***
Y27 Addition of Fractions	The reliability of this subtopic was insufficient to warrant its inclusion in the analyses.		
Y28 Subtraction of Fractions	X4 Associative Memory Plus Conversion	0.045	4.456***
	X5 Reasoning -- Number Relations	0.021	2.595**
	X6 Verbal Meaning -- Synonyms	0.047	4.398***
Y29 Multiplication of Fractions	Since this subtopic was not taught at this grade level, it was not included in the analyses.		
Y30 Division of Fractions	Since this subtopic was not taught at this grade level, it was not included in the analyses.		
Y31 Total Topic: Fractions	X4 Associative Memory Plus Conversion	0.062	2.984**
	X2 Memory Span	0.252	3.463***
	X5 Reasoning -- Number Relations	0.055	3.330***
	X6 Verbal Meaning -- Synonyms	0.128	5.854***
	X3 Associative Memory	0.056	2.194*

^aThe degrees of freedom for determining the probability levels associated with the t -ratios equal $n-k-1$ and where n is the number of subjects, k is the number of X variables in the regression equation. N for Grade 4 is 244.

* $p < .05$; ** $p < .01$; *** $p < .001$

Table 11

Partial Regression Coefficients and t -ratios: Grade 5^a

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y1 Addition	X4 Associative Memory Plus Conversion	0.048	5.187***
	X5 Reasoning -- Number Relations	0.032	5.276***
Y2 Subtraction	X6 Verbal Meaning -- Synonyms	0.035	3.846***
	X4 Associative Memory Plus Conversion	0.030	2.540*
Y3 Multiplication	X5 Reasoning -- Number Relations	0.043	5.584***
	X8 Reasoning -- Word Grouping	0.060	3.423***
Y4 Division	X5 Reasoning -- Number Relations	0.044	5.761***
	X8 Reasoning -- Word Grouping	0.058	3.401***
Y5 Laws and Generalizations	X5 Reasoning -- Number Relations	0.021	2.983**
	X6 Verbal Meaning -- Synonyms	0.028	2.620**
	X8 Reasoning -- Word Grouping	0.035	2.087*
Y6 Total Topic: Number, Operations, Assumptions	X4 Associative Memory Plus Conversion	0.102	3.636***
	X5 Reasoning -- Number Relations	0.129	6.708***
	X6 Verbal Meaning -- Synonyms	0.097	3.387***
	X8 Reasoning -- Word Grouping	0.158	3.476***

Table 11 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_1	t for b_1
Y7 Definition of Terms	The reliability of this subtopic was insufficient to warrant inclusion in the analyses.		
	X2 Memory Span	-0.067	1.997*
	X5 Reasoning -- Number Relations	0.030	4.178***
Y8 Recognition of Geometric Figures	X6 Verbal Meaning -- Synonyms	0.036	3.398***
	X7 Spatial Relations -- Visualization	0.035	2.445*
Y9 Measurement of Geometric Figures	X5 Reasoning -- Number Relations	0.037	4.767***
	X8 Reasoning -- Word Grouping	0.067	3.840***
	X5 Reasoning -- Number Relations	0.074	4.623***
Y10 Total Topic: Geometry	X6 Verbal Meaning -- Synonyms	0.067	2.806**
	X7 Spatial Relations -- Visualization	0.068	2.229*
	X8 Reasoning -- Word Grouping	0.078	2.104*
	X5 Reasoning -- Number Relations	0.034	4.784***
Y11 Equality	X6 Verbal Meaning -- Synonyms	0.031	2.910**
	X8 Reasoning -- Word Grouping	0.037	2.216*
Y12 Order Relations	X5 Reasoning -- Number Relations	0.043	6.409***
	X6 Verbal Meaning -- Synonyms	0.063	6.154***

Table 11 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_1	t for b_1
Y13 Maps, Graphs, Charts	X6 Verbal Meaning -- Synonyms	0.043	3.952***
	X8 Reasoning -- Word Grouping	0.055	3.412***
Y14 Functions	X2 Memory Span	0.092	2.572*
	X5 Reasoning -- Number Relations	0.035	4.540***
	X8 Reasoning -- Word Grouping	0.089	5.170***
Y15 Total Topic: Relations	X5 Reasoning -- Number Relations	0.118	7.483***
	X6 Verbal Meaning -- Synonyms	0.137	5.735***
	X8 Reasoning -- Word Grouping	0.185	5.013***
Y16 Sets	X6 Verbal Meaning -- Synonyms	0.044	4.056***
	X7 Spatial Relations -- Visualization	0.043	2.721**
Y17 Number Bases	X5 Reasoning -- Number Relations	0.055	6.322***
	X6 Verbal Meaning -- Synonyms	0.045	3.351***
Y18 Reading, Writing, Using Numbers	X1 Perceptual Discrimination -- Sets of Digits	0.031	2.785**
	X5 Reasoning -- Number Relations	0.031	4.432***
	X6 Verbal Meaning -- Synonyms	0.046	4.315***
	X8 Reasoning -- Word Grouping	0.046	2.781**

Table 11 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y19 Roman Numerals	X5 Reasoning -- Number Relations	0.044	5.451***
	X6 Verbal Meaning -- Synonyms	0.065	5.260***
Y20 Total Topic: Numeration	X5 Reasoning -- Number Relations	0.131	6.680***
	X6 Verbal Meaning -- Synonyms	0.169	5.694***
	X8 Reasoning -- Word Grouping	0.156	3.406***
Y21 Concepts of Measurement	X5 Reasoning -- Number Relations	0.022	2.568*
	X6 Verbal Meaning -- Synonyms	0.034	3.636**
	X7 Spatial Relations -- Visualization	0.040	2.458*
	X8 Reasoning -- Word Grouping	0.051	2.555*
Y22 Conversion of Units	X5 Reasoning -- Number Relations	0.035	4.875***
	X6 Verbal Meaning -- Synonyms	0.045	4.085***
	X8 Reasoning -- Word Grouping	0.037	2.201*
Y23 Operations - No Conversion of Units	X5 Reasoning -- Number Relations	0.022	3.277***
	X6 Verbal Meaning -- Synonyms	0.038	3.737***
	X8 Reasoning -- Word Grouping	0.059	3.754***

Table 11 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y24 Operations - Conversion of Units	X5 Reasoning -- Number Relations	0.056	8.207
	X7 Spatial Relations -- Visualization	0.049	3.177**
Y25 Total Topic: Measurement	X5 Reasoning -- Number Relations	0.125	6.703***
	X6 Verbal Meaning -- Synonyms	0.123	4.418***
	X7 Spatial Relations -- Visualization	0.130	3.680***
	X8 Reasoning -- Word Grouping	0.147	3.427***
Y26 Concepts of Fractions	X2 Memory Span	-0.097	2.827**
	X5 Reasoning -- Number Relations	0.046	7.819***
Y27 Addition of Fractions	X5 Reasoning -- Number Relations	0.042	6.708***
	X8 Reasoning -- Word Grouping	0.074	5.263**
Y28 Subtraction of Fractions	X5 Reasoning -- Number Relations	0.071	11.900***
Y29 Multiplication of Fractions	Since this subtopic was not taught at this grade level, it was excluded from the analyses.		
Y30 Division of Fractions	Since this subtopic was not taught at this grade level, it was excluded from the analyses.		

Table 11 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y31 Operations With Fractions	X2 Memory Span	-0.139	2.223*
	X4 Associative Memory Plus Conversion	0.044	2.083*
	X5 Reasoning -- Number Relations	0.115	7.895***
	X6 Verbal Meaning -- Synonyms	0.539	2.543*
	X7 Spatial Relations -- Visualization	0.797	2.902**
	X8 Reasoning -- Word Grouping	0.785	2.329*

^aThe degrees of freedom for determining the probability levels associated with the t -ratios equal $n-k-1$ and where n is the number of subjects, k is the number of X variables in the regression equation. N for Grade 5 is 261.

* $P < .05$; ** $P < .01$; *** $P < .001$

Table 12

Partial Regression Coefficients and t -ratios: Grade 6^a

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y1 Addition	X4 Associative Memory Plus Conversion	0.014	2.630**
	X1 Perceptual Discrimination -- Sets of Digits	0.033	2.710**
Y2 Subtraction	X6 Verbal Meaning -- Synonyms	0.058	6.198***
	X1 Perceptual Discrimination -- Sets of Digits	0.030	2.168*
Y3 Multiplication	X3 Associative Memory	0.040	2.820**
	X5 Reasoning -- Number Relations	0.026	3.111**
	X6 Verbal Meaning -- Synonyms	0.034	2.350*
	X8 Reasoning -- Word Grouping	0.046	2.147*
	X4 Associative Memory Plus Conversion	0.022	3.169**
Y4 Division	X1 Perceptual Discrimination -- Sets of Digits	0.056	3.361***
	X6 Verbal Meaning -- Synonyms	0.055	4.077***
Y5 Laws and Generalizations	X5 Reasoning -- Number Relations	0.023	3.303***
	X6 Verbal Meaning -- Synonyms	0.025	2.356*

Table 12 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y6 Total Topic: Number, Operations, Assumptions	X4 Associative Memory Plus Conversion	0.036	2.086*
	X1 Perceptual Discrimination -- Sets of Digits	0.114	2.765**
	X5 Reasoning -- Number Relations	0.071	2.925**
	X6 Verbal Meaning -- Synonyms	0.126	3.044**
	X8 Reasoning -- Word Grouping	0.127	2.063*
Y7 Definition of Terms	X5 Reasoning -- Number Relations	0.030	3.355***
	X6 Verbal Meaning -- Synonyms	0.035	2.551*
Y8 Recognition of Geometric Figures	X6 Verbal Meaning -- Synonyms	0.068	5.169***
	X7 Spatial Relations -- Visualization	0.030	2.342*
	X8 Reasoning -- Word Grouping	0.042	2.065*
Y9 Measurement of Geometric Figures	X4 Associative Memory Plus Conversion	0.013	2.076*
	X5 Reasoning -- Number Relations	0.036	4.049***
	X6 Verbal Meaning -- Synonyms	0.040	2.584**
	X7 Spatial Relations - Visualization	0.030	2.085*
	X8 Reasoning -- Word Grouping	0.065	2.754**

Table 12 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y10 Total Topic: Geometry	X4 Associative Memory Plus Conversion	0.032	2.640**
	X5 Reasoning -- Number Relations	0.068	3.844***
	X6 Verbal Meaning -- Synonyms	0.118	4.018***
	X7 Spatial Relations -- Visualization	0.070	2.583**
	X8 Reasoning -- Word Grouping	0.118	2.637**
Y11 Equality	X5 Reasoning -- Number Relations	0.036	4.878***
	X7 Spatial Relations -- Visualization	0.025	1.975*
	X8 Reasoning -- Word Grouping	0.050	2.845**
Y12 Order Relations	X5 Reasoning -- Number Relations	0.047	4.897***
	X6 Verbal Meaning -- Synonyms	0.066	3.960***
	X8 Reasoning -- Word Grouping	0.064	2.533**
Y13 Maps, Graphs, Charts	X5 Reasoning -- Number Relations	0.024	2.967**
	X5 Verbal Meaning -- Synonyms	0.041	3.189**
Y14 Functions	X1 Perceptual Discrimination -- Sets of Digits	0.049	2.992**
	X5 Reasoning -- Number Relations	0.027	2.908**
	X8 Reasoning -- Word Grouping	0.119	5.555***

Table 12 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y15 Total Topic: Relations	X5 Reasoning -- Number Relations	0.129	6.181***
	X6 Verbal Meaning -- Synonyms	0.142	3.891***
	X7 Spatial Relations -- Visualization	0.079	2.285*
	X8 Reasoning -- Word Grouping	0.195	3.442***
Y16 Sets	X5 Reasoning -- Number Relations	0.032	4.079***
Y17 Number Bases	X5 Reasoning -- Number Relations	0.037	3.283***
	X8 Reasoning -- Word Grouping	0.088	3.374***
Y18 Reading, Writing, Using Numbers	X5 Reasoning -- Number Relations	0.036	4.395***
	X6 Verbal Meaning -- Synonyms	0.053	3.634***
	X8 Reasoning -- Word Grouping	0.077	3.515***
Y19 Roman Numerals	X3 Associative Memory	0.041	2.425*
	X5 Reasoning -- Number Relations	0.032	3.207**
	X8 Reasoning -- Word Grouping	0.101	4.515***

Table 12 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y20 Total Topic: Numeration	X5 Reasoning -- Number Relations	0.123	5.637***
	X6 Verbal Meaning -- Synonyms	0.126	3.311***
	X7 Spatial Relations -- Visualization	0.082	2.298*
	X8 Reasoning -- Word Grouping	0.211	3.570***
Y21 Concepts of Measurement	X5 Reasoning -- Number Relations	0.022	2.274*
	X6 Verbal Meaning -- Synonyms	0.063	3.739***
	X8 Reasoning -- Word Grouping	0.076	2.982**
Y22 Conversion of Units	X3 Associative Memory	0.035	2.320*
	X5 Reasoning -- Number Relations	0.041	4.811***
	X6 Verbal Meaning -- Synonyms	0.059	4.495***
Y23 Operations - No Conversion of Units	X1 Perceptual Discrimination -- Sets of Digits	0.034	3.046**
	X2 Memory Span	-0.065	2.258*
	X5 Reasoning -- Number Relations	0.024	3.706***
	X6 Verbal Meaning -- Synonyms	0.032	3.283***
Y24 Operations - Conversion of Units	X5 Reasoning -- Number Relations	0.038	4.460***
	X6 Verbal Meaning -- Synonyms	0.052	3.485***
	X8 Reasoning -- Word Grouping	0.056	2.493*

Table 12 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_i	t for b_i
Y25 Total Topic: Measurement	X1 Perceptual Discrimination -- Sets of Digits	0.096	2.497*
	X5 Reasoning -- Number Relations	0.115	5.049***
	X6 Verbal Meaning -- Synonyms	0.185	4.711***
	X8 Reasoning -- Word Grouping	0.200	3.358***
Y26 Concepts of Fractions	X2 Memory Span	-0.079	2.430*
	X5 Reasoning -- Number Relations	0.019	2.488*
	X6 Verbal Meaning -- Synonyms	0.049	3.795***
	X7 Spatial Relations -- Visualization	0.028	2.251*
	X8 Reasoning -- Word Grouping	0.069	3.391***
Y27 Addition of Fractions	X3 Associative Memory	0.026	2.019*
	X5 Reasoning -- Number Relations	0.021	2.791**
	X8 Reasoning -- Word Grouping	0.111	6.491***
Y28 Subtraction of Fractions	X4 Associative Memory Plus Conversion	0.012	2.049*
	X5 Reasoning -- Number Relations	0.025	3.111**
	X6 Verbal Meaning -- Synonyms	0.075	5.969***

Table 12 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	b_1	t for b_1
Y29 Multiplication of Fractions	X4 Associative Memory Plus Conversion	0.020	3.362***
	X6 Verbal Meaning -- Synonyms	0.056	3.810***
	X7 Spatial Relations -- Visualization	0.030	2.173*
	X8 Reasoning -- Word Grouping	0.073	3.305***
Y30 Division of Fractions	X4 Associative Memory Plus Conversion	0.020	3.151**
	X6 Verbal Meaning -- Synonyms	0.068	5.158***
Y31 Operations - With Fractions	X4 Associative Memory Plus Conversion	0.063	3.489***
	X5 Reasoning -- Number Relations	0.075	2.911**
	X6 Verbal Meaning -- Synonyms	0.244	5.548***
	X7 Spatial Relations -- Visualization	0.090	2.198*
	X8 Reasoning -- Word Grouping	0.245	3.647***

^aThe degrees of freedom for determining the probability levels associated with the t -ratios equals $n-k-1$, where n is the number of subjects, and k is the number of X variables in the regression equation. N for Grade 6 is 240.

* $p < .05$; ** $p < .01$; *** $p < .001$

Research Objective 2
Extent of the Contributions by the Intellectual Processes
to the Mathematics Topics and Subtopics

The second research objective was to determine to what extent each identified intellectual process, or combination of intellectual processes, is related to achievement of the instructional objectives for each selected mathematics topic and subtopic. The data which are most immediately relevant to this objective are the correlation coefficients and the corresponding coefficients of determination computed in the step-wise regression analyses (Appendix H) to reflect the intellectual processes which contributed significantly to explaining the total score variance for each mathematics topic or subtopic. When only one intellectual process contributed, simple correlation coefficients and their corresponding coefficients of determination were pertinent; when more than one intellectual process contributed, multiple correlation coefficients and their corresponding coefficients of determination were pertinent. These data are presented in Tables 13, 14, and 15. Table 13 contains data for the fourth grade level; Table 14, for the fifth grade level; and Table 16, for the sixth grade level.

Table 13

Multiple Correlation Coefficients and
Coefficients of Determination % Grade 4^a

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
Y1 Addition	X1 Perceptual Discrimination -- Sees of Digits	.252	6.37
	X3 Associative Memory		
Y2 Subtraction	X3 Associative Memory	.487	23.69
	X5 Reasoning - Number Relations		
Y3 Multiplication	X6 Verbal Meaning Synonyms	.490	24.04
	X5 Reasoning - Number Relations		
Y4 Division	X6 Verbal Meaning - Synonyms	.400	15.99
	X5 Reasoning - Number Relations		
Y5 Laws and Generalizations	X7 Spatial Relations - Visualization	.504	25.43
	X5 Reasoning - Number Relations		
Y6 Total topic: Number, Operations, Assumptions	X6 Verbal Meaning - Synonyms	.630	39.39
	X3 Associative Memory		

Table 13 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in)%
Y7 Definitions of Terms	-----	----	----
Y8 Recognition of geometric figures	X6 Verbal Meaning - Synonyms	.350	12.18
Y9 Measurement of geometric figures	-----	----	----
Y10 Total Topic: Geometry	X1 Perceptual Discrimination- Sets of Digits		
	X2 Memory Span	.357	12.66
	X6 Verbal Meaning - Synonyms		
Y11 Equality	X2 Memory Span		
	X5 Reasoning - Number Relations	.591	34.95
	X6 Verbal Meaning - Synonyms		
Y12 Order Relations	X1 Perceptual Discrimination - Sets of Digits	.360	12.93
	X6 Verbal Meaning - Synonyms		
Y13 Maps, Graphs, Charts	X5 Reasoning - Number Relations	.363	13.14
	X6 Verbal Meaning - Synonyms		
Y14 Functions	-----	----	----
Y15 Total Topic: Relations	X3 Associative Memory		
	X5 Reasoning - Number Relations	.635	40.37
	X6 Verbal Meaning - Synonyms		

Table 13 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
Y16 Sets	X5 Reasoning - Number Relations	.468	22.00
	X4 Associative Memory Plus Conversion		
Y17 Number Base	X1 Perceptual Discrimination - Sets of Digits	.497	24.68
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		
Y18 Reading, Writing, Using Numbers	X5 Reasoning - Number Relations	.560	31.46
	X4 Associative Memory Plus Conversion		
Y19 Roman Numerals	X2 Memory Span	.535	28.59
	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms		
	X4 Associative Memory Plus Conversion		
	X1 Perceptual Discrimination - Sets of Digits		
Y20 Total Topic: Numeration.	X5 Reasoning - Number Relations	.742	54.99
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		

Table 13 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
Y21 Concepts of Measurement	X4 Associative Memory Plus Conversion		
	X2 Memory Span		
	X5 Reasoning - Number Relations	.462	21.33
	X6 Verbal Meaning - Synonyms		
Y22 Conversion of Units	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.515	26.47
Y23 Operations No Conversion	X4 Associative Memory Plus Conversion		
	X2 Memory Span		
	X5 Reasoning - Number Relations	.641	41.03
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		
Y24 Operations with Conversion	X2 Memory Span		
	X5 Reasoning - Number Relations	.487	23.75
	X6 Verbal Meaning - Synonyms		
Y25 Total Topic: Measurment	X4 Associative Memory Plus Conversion		
	X2 Memory Span		
	X5 Reasoning - Number Relations	.702	49.30
	X6 Verbal Meaning - Synonyms		

Table 13 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
Y26 Concepts	X1 Perceptual Discrimination - Sets of Digits		
	X2 Memory Span	.447	19.96
	X3 Associative Memory		
	X6 Verbal Meaning - Synonyms		
Y27 Addition	-----	----	----
Y28 Subtraction	X4 Associative Memory Plus Conversion		
	X5 Reasoning - Number Relations	.559	31.24
	X6 Verbal Meaning - Synonyms		
Y29 Multiplication	-----	----	----
Y30 Division	-----	----	----
Y31 Total Topic: Fractions	X4 Associative Memory Plus Conversion		
	X2 Memory Span		
	X3 Associative Memory	.669	44.74
	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms		

^aSee section on Analyses for explanation of deleted statistics.

Table 14

**Multiple Correlation Coefficients and
Coefficients of Determination: Grade 5^a**

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
Y1 addition	X4 Associative Memory Plus Conversion	.307	9.41
Y2 Subtraction	X5 Reasoning - Number Relations	.512	26.18
	X6 Verbal Meaning - Synonyms		
Y3 Multiplication	X4 Associative Memory Plus Conversion		
	X5 Reasoning - Number Relations	.602	36.21
	X8 Reasoning - Word Grouping		
Y4 Division	X5 Reasoning - Number Relations	.545	29.67
	X8 Reasoning - Word Grouping		
Y5 Laws and Generalizations	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.474	22.45
	X8 Reasoning - Word Grouping		
Y6 Total Topic: Number, Operations, Assumptions	X4 Associative Memory Plus Conversion		
	X5 Reasoning - Number Relations	.475	55.48
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		

Table 14 (Continued)

Y7 Definitions of Terms	-----	----	----
	X2 Memory Span		
Y8 Recognition of Geometric Figures	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.528	27.87
	X7 Spatial Relations - Visualization		
Y9 Measurement of Geometric Figures	X5 Reasoning - Number Relations	.518	26.87
	X8 Reasoning - Word Grouping		
Y10 Total Topic: Geometry	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.611	37.33
	X7 Spatial Relations - Visualization		
	X8 Reasoning - Word Grouping		
Y11 Equality	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.572	32.74
	X8 Reasoning - Word Grouping		
Y12 Order Relations	X5 Reasoning - Number Relations	.633	40.05
	X6 Verbal Meaning - Synonyms		

Table 14 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
Y13 Maps, Graphs Charts	X6 Verbal Meaning - Synonyms	.452	20.39
	X8 Reasoning - Word Grouping		
Y14 Functions	X2 Memory Span	.579	33.57
	X5 Reasoning - Number Relations		
	X8 Reasoning - Word Grouping		
Y15 Total Topic: Relations	X5 Reasoning - Number Relations	.788	62.01
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		
Y16 Sets	X6 Verbal Meaning - Synonyms	.354	12.53
	X7 Spatial Relations - Visualization		
Y17 Number Bases	X5 Reasoning - Number Relations	.538	28.93
	X6 Verbal Meaning - Synonyms		
Y18 Reading, Writing Using Numbers	X1 Perceptual Discrimination Sets of Digits	.647	41.79
	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		

Table 14 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
Y19 Roman Numerals	X5 Reasoning - Number Relations	.572	32.69
	X6 Verbal Meaning - Synonyms		
Y20 Total Topic: Numeration	X5 Reasoning - Number Relations	.741	52.92
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		
Y21 Concepts of Measurement	X5 Reasoning - Number Relations	.547	29.91
	X6 Verbal Meaning - Synonyms		
	X7 Spatial Relations Visualization		
	X8 Reasoning - Word Grouping		
Y22 Conversion of Units	X5 Reasoning - Number Relations	.616	37.90
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		
Y23 Operations No Conversion	X5 Reasoning - Number Relations	.603	36.37
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		

Table 14 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in%)
Y24 Operations with Conversion	X5 Reasoning - Number Relations	.568	32.26
	X7 Spatial Relations - Visualization		
Y25 Total Topic: Measurement	X5 Reasoning - Number Relations	.765	58.51
	X6 Verbal Meaning - Synonyms		
	X7 Spatial Relations - Visualization		
	X8 Reasoning - Word Grouping		
Y26 Concepts	X2 Memory Span	.452	20.40
	X5 Reasoning - Number Relations		
Y27 Addition	X5 Reasoning - Number Relations	.645	41.57
	X8 Reasoning - Word Grouping		
Y28 Subtraction	X5 Reasoning - Number Relations	.623	38.77
Y29 Multiplication	-----	--	--
Y30 Division	-----	--	--

Table 14 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in%)
	X4 Associative Memory Plus Conversion		
	X2 Memory Span		
	X5 Reasoning - Number Relations		
Y31 Total Topic: Fractions	X6 Verbal Meaning - Synonyms	.757	57.26
	X7 Spatial Relations - Visualization		
	X8 Reasoning - Word Grouping		

^aSee section on Analyses for explanation of deleted statistics.

Table 15

**Multiple Correlation Coefficients and
Coefficients of Determination: Grade 6^a**

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
Y1 Addition	X4 Associative Memory Plus Conversion	.168	2.82
Y2 Subtraction	X1 Perceptual Discrimination - Sets of Digits	.433	18.76
	X6 Verbal Meaning - Synonyms		
	X1 Perceptual Discrimination - Sets of Digits		
	X3 Associative Memory		
Y3 Multiplication	X5 Reasoning - Number Relations	.604	36.47
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		
Y4 Division	X4 Associative Memory Plus Conversion		
	X1 Perceptual Discrimination - Sets of Digits	.498	24.80
	X6 Verbal Meaning - Synonyms		
Y5 Laws and Generalizations	X5 Reasoning - Number Relations	.350	12.24
	X6 Verbal Memory - Synonyms		

Table 15 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
	X4 Associative Memory Plus Conversion		
	X1 Perceptual Discrimination - Sets of Digits		
Y6 Total Topic: Number, Operations Assumptions	X5 Reasoning - Number Relations	.616	37.97
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		
Y7 Definitions of Terms	-----	----	----
	X6 Verbal Meaning - Synonyms		
Y8 Recognition of Geometric Figures	X7 Spatial Relations - Visualization	.553	30.62
	X8 Reasoning - Word Grouping		
	X4 Associative Memory Plus Conversion		
Y9 Measurement of Geometric Figures	X5 Reasoning - Number Relations	.653	42.70
	X6 Verbal Meaning - Synonyms		
	X7 Spatial Relations - Visualization		
	X8 Reasoning - Word Grouping		

Table 15 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
Y10 Total Topic: Geometry	X4 Associative Memory Plus Conversion		
	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.706	49.79
	X7 Spatial Relations - Visualization		
Y11 Equality	X8 Reasoning - Word Grouping		
	X5 Reasoning - Number Relations		
	X7 Spatial Relations - Visualization	.516	26.62
Y12 Order Relations	X8 Reasoning - Word Grouping		
	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.634	40.17
Y13 Maps, Graphs, Charts	X8 Reasoning - Word Grouping		
	X5 Reasoning - Number Relations	.375	14.07
X14 Functions	X6 Verbal Meaning - Synonyms		
	X1 Perceptual Discrimination - Sets of Digits		
	X5 Reasoning - Number Relations	.553	30.63
	X8 Reasoning - Word Grouping		

Table 15 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in%)
Y15 Total Topic: Relations	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.726	52.69
	X7 Spatial Relations - Visualization		
	X8 Reasoning - Word Grouping		
Y16 Sets	X5 Reasoning - Number Relations	.256	6.53
Y17 Number Bases	X5 Reasoning - Number Relations	.407	16.60
	X8 Reasoning - Word Grouping		
Y18 Reading, Writing, Using Numbers	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.641	41.13
	X8 Reasoning - Word Grouping		
Y19 Roman Numerals	X3 Associative Memory		
	X5 Reasoning - Number Relations	.537	28.85
	X8 Reasoning - Word Grouping		
Y20 Total Topic: Numeration	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.702	49.34
	X7 Spatial Relations - Visualization		
	X8 Reasoning - Word Grouping		

Table 15 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in%)
Y21 Concepts of Measurement	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.557	31.02
	X8 Reasoning - Word Grouping		
Y22 Conversion of Units	X3 Associative Memory		
	X5 Reasoning - Number Relations	.595	35.37
	X6 Verbal Meaning - Synonyms		
Y23 Operations No Conversion	X1 Perceptual Discrimination - Sets of Digits		
	X2 Memory Span		
	X5 Reasoning - Number Relations	.493	24.33
	X6 Verbal Meaning - Synonyms		
Y24 Operations with Conversion	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.600	36.05
	X8 Reasoning - Word Grouping		
Y25 Total Topic Measurement	X1 Perceptual Discrimination - Sets of Digits		
	X5 Reasoning - Number Relations	.713	50.84
	X6 Verbal Meaning - Synonyms		
	X8 Reasoning - Word Grouping		

Table 15 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
Y26 Concepts	X2 Memory Span		
	X5 Reasoning - Number Relations		
	X6 Verbal Meaning - Synonyms	.632	39.96
	X7 Spatial Relations - Visualization		
	X8 Reasoning - Word Grouping		
Y27 Addition	X3 Associative Memory		
	X5 Reasoning - Number Relations	.590	34.75
	X8 Reasoning - Word Grouping		
Y28 Subtraction	X4 Associative Memory Plus Conversion		
	X5 Reasoning - Number Relations	.586	34.34
	X6 Verbal Meaning - Synonyms		
Y29 Multiplication	X4 Associative Memory Plus Conversion		
	X6 Verbal Meaning - Synonyms	.627	39.27
	X7 Spatial Relations - Visualization		
	X8 Reasoning - Word Grouping		
Y30 Division	X4 Associative Memory Plus Conversion	.458	20.93
	X6 Verbal Meaning - Synonyms		

Table 15 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	R	R ² (in %)
	X4 Associative Memory Plus Conversion		
	X5 Reasoning - Number Relations		
Y31 Total Topic: Fractions	X6 Verbal Meaning - Synonyms	.759	57.54
	X7 Spatial Relations - Visualization		
	X8 Reasoning - Word Grouping		

^aSee section on Analyses for explanation of delated statistics.

Research Objective 3

Relative Contributions by the Intellectual Processes: Comparisons Within Each Mathematics Topic and Subtopic

The third research objective was to specify the relative contribution, or weighting, of each intellectual process when combinations of intellectual processes are related to achievement of the instructional objectives for each selected mathematics topic and subtopic. Pertinent data include the standard partial regression coefficients (B)¹ obtained in the step-wise regression analyses (Appendix H) and the percentage contribution² of each intellectual process entering the regression equation. These data are presented in Tables 16, 17, and 18. Table 16 consists of data for the fourth grade level; Table 17, for the fifth grade level; Table 18, for the sixth grade level.

¹The symbol, B, is used hereafter in the text and the tables to refer to the standard partial regression coefficient.

²Refer to the Analyses section in Chapter II for a description of how the percentage contribution of each variable was computed when more than one intellectual process entered the regression equation. When only one intellectual process entered the regression equation, the r^2 represents the percentage contribution.

Table 16.

**Standard Partial Regression Coefficients and Corresponding
Percentage Contributions; Grade 4^a**

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	% Contri- bution
Y1 Addition	X1 Perceptual Discrimination -- Sets of Digits	.156	2.77
	X3 Associative Memory	.181	3.60
Y2 Subtraction	X3 Associative Memory	.136	3.44
	X5 Reasoning -- Number Relations	.276	11.99
	X6 Verbal Meaning -- Synonyms	.208	8.26
Y3 Multiplication	X5 Reasoning -- Number Relations	.308	13.83
	X6 Verbal Meaning -- Synonyms	.242	10.21
Y4 Division	X5 Reasoning -- Number Relations	.324	12.32
	X7 Spatial Relations -- Visualization	.136	3.67
Y5 Laws and Generalizations	X5 Reasoning -- Number Relations	.309	14.17
	X6 Verbal Meaning -- Synonyms	.257	11.26

Table 16 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	% Contri- bution
	X3 Associative Memory	.104	2.79
Y6 Total Topic: Number, Operations Assumptions	X5 Reasoning -- Number Relations	.406	23.68
	X6 Verbal Meaning -- Synonyms	.255	13.12
Y7 Definition of Geometric Terms	-----	----	----
Y8 Recognition of Geometric Figures	X6 Verbal Meaning -- Synonyms	.349	12.18
Y9 Measurement of Geometric Figures	-----	----	----
	X1 Perceptual Discrimination -- Sets of Digits	.167	2.95
Y10 Total Topic: Geometry	X2 Memory Span	-.134	.69
	X6 Verbal Meaning -- Synonyms	.298	9.02
	X2 Memory Span	.152	2.53
Y11 Equality	X5 Reasoning -- Number Relations	.384	20.07
	X6 Verbal Meaning -- Synonyms	.250	12.36
Y12 Order Relations	X1 Perceptual Discrimination -- Sets of Digits	.120	1.99
	X6 Verbal Meaning -- Synonyms	.323	10.95
Y13 Maps, Graphs, Charts	X5 Reasoning -- Number Relations	.176	5.45

Table 16 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B ₁	% Contri- bution
Y13 Maps, Graphs, Charts (Continued)	X6 Verbal Meaning -- Synonyms	.231	7.69
Y14 Functions	-----	----	----
	X3 Associative Memory	.122	3.37
Y15 Total Topic: Relations	X5 Reasoning -- Number Relations	.266	14.11
	X6 Verbal Meaning -- Synonyms	.397	22.89
Y16 Sets	X5 Reasoning -- Number Relations	.471	22.04
	X4 Associative Memory Plus Conversion	.158	5.18
Y17 Number Bases	X1 Perceptual Discrimination -- Sets of Digits	.125	3.16
	X6 Verbal Meaning -- Synonyms	.214	8.39
	X8 Reasoning -- Word Grouping	.201	7.95
Y18 Reading, Writing, Using Numbers	X5 Reasoning -- Number Relations	.563	31.46
	X4 Associative Memory Plus Conversion	.138	4.34
Y19 Roman Numerals	X2 Memory Span	.120	1.97
	X5 Reasoning -- Number Relations	.260	11.58
	X6 Verbal Meaning -- Synonyms	.241	10.70

Table 16 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	% Contri- bution
Y20 Total Topic: Numeration	X4 Associative Memory Plus Conversion	.170	7.63
	X1 Perceptual Discrimination -- Sets of Digits	.138	4.49
	X5 Reasoning -- Number Relations	.254	15.43
	X6 Verbal Meaning -- Synonyms	.309	18.91
	X8 Reasoning -- Word Grouping	.155	8.53
Y21 Concepts of Measurement	X4 Associative Memory Plus Conversion	.124	3.45
	X2 Memory Span	.146	2.62
	X5 Reasoning -- Number Relations	.228	8.51
	X6 Verbal Meaning -- Synonyms	.183	6.74
Y22 Conversion of of Units	X5 Reasoning -- Number Relations	.337	16.05
	X6 Verbal Meaning -- Synonyms	.239	10.41
Y23 Operations - No Conversion	X4 Associative Memory Plus Conversion	.209	8.69
	X2 Memory Span	.120	2.14
	X5 Reasoning -- Number Relations	.243	12.49
	X6 Verbal Meaning -- Synonyms	.194	9.75

Table 16 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	% Contri- bution
Y23 Operations - No Conversion (Continued)	X8 Reasoning -- Word Grouping	.165	7.96
	X2 Memory Span	.120	1.80
Y24 Operations - With Conversion	X5 Reasoning -- Number Relations	.222	8.66
	X6 Verbal Meaning -- Synonyms	.299	13.29
	X4 Associative Memory Plus Conversion	.134	4.97
	X2 Memory Span	.129	2.32
Y25 Total Topic: Measurement	X5 Reasoning -- Number Relations	.360	21.63
	X6 Verbal Meaning -- Synonyms	.339	20.38
	X1 Perceptual Discrimination -- Sets of Digits	.157	3.89
Y26 Concepts	X2 Memory Span	.163	3.95
	X3 Associative Memory	.121	2.42
	X6 Verbal Meaning -- Synonyms	.278	9.70
Y27 Addition	-----	----	----
Y28 Subtraction	X4 Associative Memory Plus Conversion	.257	10.12
	X5 Reasoning -- Number Relations	.178	7.87

Table 16 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	% Contri- bution
Y28 Subtraction (Continued)	X6 Verbal Meaning -- Synonyms	.290	13.26
Y29 Multiplication	-----	----	----
Y30 Division	-----	----	----
	X4 Associative Memory Plus Conversion	.162	6.33
	X2 Memory Span	.174	4.23
Y31 Total Topic: Fractions	X3 Associative Memory	.113	3.34
	X5 Reasoning -- Number Relations	.212	10.61
	X6 Verbal Meaning -- Synonyms	.356	20.23

^aSee section on Analyses for explanation of deleted statistics.

Table 17

Standard Partial Regression Coefficients and Corresponding
Percentage Contributions β_i -- Grade 5^a

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	% Contri- bution
Y1 Addition	X4 Associative Memory Plus Conversion	.307	9.41
Y2 Subtraction	X5 Reasoning -- Number Relations	.336	15.73
	X6 Verbal Meaning -- Synonyms	.245	10.45
Y3 Multiplication	X4 Associative Memory Plus Conversion	.143	5.44
	X5 Reasoning -- Number Relations	.357	19.63
	X8 Reasoning -- Word Grouping	.222	11.14
Y4 Division	X5 Reasoning -- Number Relations	.379	19.51
	X8 Reasoning -- Word Grouping	.224	10.16
Y5 Laws and Generalizations	X5 Reasoning -- Number Relations	.216	8.87
	X6 Verbal Meaning -- Synonyms	.186	7.34
	X8 Reasoning -- Word Grouping	.157	6.23
Y6 Total Topic: Number, Operations Assumptions	X4 Associative Memory Plus Conversion	.171	7.99
	X5 Reasoning -- Number Relations	.374	24.83

Table 17 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_1	% Contri- bution
Y6 Total Topic: Number, Operations, Assumptions (Continued)	X6 Verbal Meaning -- Synonyms	.183	10.22
	X8 Reasoning -- Word Grouping	.204	12.45
Y7 Definition of Terms	-----	----	----
Y8 Recognition of Geometric Figures	X2 Memory Span	-.107	.96
	X5 Reasoning -- Number Relations	.278	12.55
	X6 Verbal Meaning -- Synonyms	.218	9.27
	X7 Spatial Relations -- Visualization	.146	5.09
Y9 Measurement of Geometric Figures	X5 Reasoning -- Number Relations	.320	15.23
	X8 Reasoning -- Word Grouping	.258	11.64
Y10 Total Topic: Geometry	X5 Reasoning -- Number Relations	.308	16.89
	X6 Verbal Meaning -- Synonyms	.181	8.64
	X7 Spatial Relations -- Visualization	.125	4.80
	X8 Reasoning -- Word Grouping	.144	7.02
Y11 Equality	X5 Reasoning -- Number Relations	.322	16.74
	X6 Verbal Meaning -- Synonyms	.192	8.81

Table 17 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	% Contri- bution
Y11 Equality (Continued)	X8 Reasoning -- Word Grouping	.155	7.19
Y12 Order Relations	X5 Reasoning -- Number Relations	.368	20.55
	X6 Verbal Meaning -- Synonyms	.353	19.50
Y13 Maps, Graphs, Charts	X6 Verbal Meaning -- Synonyms	.272	11.13
	X8 Reasoning -- Word Grouping	.235	9.25
Y14 Functions	X2 Memory Span	.132	1.79
	X5 Reasoning -- Number Relations	.294	14.90
	X8 Reasoning -- Word Grouping	.334	16.88
Y15 Total Topic: Relations	X5 Reasoning -- Number Relations	.378	26.21
	X6 Verbal Meaning -- Synonyms	.285	18.37
	X8 Reasoning -- Word Grouping	.264	17.43
Y16 Sets	X6 Verbal Meaning -- Synonyms	.254	8.04
	X7 Spatial Relations -- Visualization	.170	4.49
Y17 Number Bases	X5 Reasoning -- Number Relations	.395	20.07

Table 17 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	% Contri- bution
Y17 Number Bases (Continued)	X6 Verbal Meaning -- Synonyms	.209	8.86
	X1 Perceptual Discrimination -- Sets of Digits	.134	2.81
Y18 Reading, Writing, Using Numbers	X5 Reasoning -- Number Relations	.278	15.11
	X6 Verbal Meaning -- Synonyms	.266	14.38
	X8 Reasoning -- Word Grouping	.182	9.49
Y19 Roman Numerals	X5 Reasoning -- Number Relations	.331	16.72
	X6 Verbal Meaning -- Synonyms	.320	15.96
Y20 Total Topic: Numeration	X5 Reasoning -- Number Relations	.368	24.04
	X6 Verbal Meaning -- Synonyms	.308	19.17
	X8 Reasoning -- Word Grouping	.195	11.71
Y21 Concepts of Measurement	X5 Reasoning -- Number Relations	.181	8.22
	X6 Verbal Meaning -- Synonyms	.179	7.89
	X7 Spatial Relations -- Visualization	.146	5.33
	X8 Reasoning -- Word Grouping	.185	8.47

Table 17 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	% Contri- bution
Y22 Conversion of Units	X5 Reasoning -- Number Relations	.315	17.20
	X6 Verbal Meaning -- Synonyms	.259	13.42
	X8 Reasoning -- Word Grouping	.148	7.28
Y23 Operations - No Conversion	X5 Reasoning -- Number Relations	.214	10.72
	X6 Verbal Meaning -- Synonyms	.240	12.18
	X8 Reasoning -- Word Grouping	.255	13.47
Y24 Operations - With Conversion	X5 Reasoning -- Number Relations	.466	25.37
	X7 Spatial Relations -- Visualization	.181	6.89
Y25 Total Topic: Measurement	X5 Reasoning -- Number Relations	.364	24.63
	X6 Verbal Meaning -- Synonyms	.231	13.93
	X7 Spatial Relations -- Visualization	.169	8.20
	X8 Reasoning -- Word Grouping	.191	11.73
Y26 Concepts	X2 Memory Span	-.158	1.95
	X5 Reasoning -- Number Relations	.436	18.45

Table 17 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	% Contri- bution
Y27 Addition	X5 Reasoning -- Number Relations	.402	23.89
	X8 Reasoning -- Word Grouping	.316	17.67
Y28 Subtraction	X5 Reasoning -- Number Relations	.379	23.57
Y29 Multiplication	-----	----	----
Y30 Division	-----	----	----
Y31 Total Topic: Fractions	X4 Associative Memory Plus Conversion	.098	4.12
	X2 Memory Span	-.093	.60
	X5 Reasoning -- Number Relations	.445	30.79
	X6 Verbal Meaning -- Synonyms	.136	7.33
	X7 Spatial Relations -- Visualization	.137	6.42
	X8 Reasoning -- Word Grouping	.136	8.01

^aSee section on Analyses for explanation on deleted statistics.

Table 18

Standard Partial Regression Coefficients and Corresponding
Percentage Contributions: Grade 6^a

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	% Contri- bution
Y1 Addition	X4 Associative Memory Plus Conversion	.168	2.82
Y2 Subtraction	X1 Perceptual Discrimination -- Sets of Digits	.162	3.83
	X6 Verbal Meaning -- Synonyms	.371	14.93
Y3 Multiplication	X1 Perceptual Discrimination -- Sets of Digits	.120	3.55
	X3 Associative Memory	.173	7.39
	X5 Reasoning -- Number Relations	.207	10.09
	X6 Verbal Meaning -- Synonyms	.172	8.18
	X8 Reasoning -- Word Grouping	.156	7.27
Y4 Division	X4 Associative Memory	.209	8.18
	X1 Perceptual Discrimination -- Sets of Digits	.203	6.66
	X6 Verbal Meaning -- Synonyms	.256	9.95
Y5 Laws and Generalizations	X5 Reasoning -- Number Relations	.234	7.47
	X6 Verbal Meaning -- Synonyms	.167	4.78

Table 18 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	% Contri- bution
Y6 Total Topic: Number, Operations, Assumptions	X4 Associative Memory Plus Conversion	.130	5.52
	X1 Perceptual Discrimination -- Sets of Digits	.154	5.03
	X5 Reasoning -- Number Relations	.192	9.29
	X6 Verbal Meaning -- Synonyms	.220	11.04
	X8 Reasoning -- Word Grouping	.148	7.09
Y7 Definition of Terms	-----	----	----
Y8 Recognition of Geometric Figures	X6 Verbal Meaning -- Synonyms	.373	19.26
	X7 Spatial Relations -- Visualization	.139	4.32
	X8 Reasoning -- Word Grouping	.155	7.04
Y9 Measurement of Geometric Figures	X4 Associative Memory Plus Conversion	.120	5.03
	X5 Reasoning -- Number Relations	.255	13.74
	X6 Verbal Meaning -- Synonyms	.179	9.40
	X7 Spatial Relations -- Visualization	.113	3.96
	X8 Reasoning -- Word Grouping	.196	10.56

Table 18 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	% Contri- bution
	X4 Associative Memory Plus Conversion	.143	6.59
	X5 Reasoning -- Number Relations	.227	12.63
Y10 Total Topic: Geometry	X6 Verbal Meaning -- Synonyms	.261	15.49
	X7 Spatial Relations -- Visualization	.132	5.01
	X8 Reasoning -- Word Grouping	.175	10.08
	X5 Reasoning -- Number Relations	.323	14.94
Y11 Equality	X7 Spatial Relations -- Visualization	.121	3.56
	X8 Reasoning -- Word Grouping	.196	8.12
	X5 Reasoning -- Number Relations	.301	16.08
Y12 Order Relations	X6 Verbal Meaning -- Synonyms	.274	14.91
	X8 Reasoning -- Word Grouping	.178	9.18
	X5 Reasoning -- Number Relations	.208	6.70
Y13 Maps, Graphs, Charts	X6 Verbal Meaning -- Synonyms	.224	7.37
Y14 Functions	X1 Perceptual Discrimination -- Sets of Digits	.169	5.02

Table 18 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	% Contri- bution
Y14 Functions (Continued)	X5 Reasoning -- Number Relations	.190	8.04
	X8 Reasoning -- Word Grouping	.356	17.57
Y15 Total Topic: Relations	X5 Reasoning -- Number Relations	.340	20.88
	X6 Verbal Meaning -- Synonyms	.240	14.24
	X7 Spatial Relations -- Visualization	.113	4.21
	X8 Reasoning -- Word Grouping	.221	13.37
Y16 Sets	X5 Reasoning -- Number Relations	.256	6.53
Y17 Number Bases	X5 Reasoning -- Number Relations	.230	8.15
	X8 Reasoning -- Word Grouping	.236	8.45
Y18 Reading, Writing, Using Numbers	X5 Reasoning -- Number Relations	.268	14.05
	X6 Verbal Meaning -- Synonyms	.249	13.64
	X8 Reasoning -- Word Grouping	.244	13.45
Y19 Roman Numerals	X3 Associative Memory	.151	5.32
	X5 Reasoning -- Number Relations	.219	9.73

Table 18 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	% Contri- bution
Y19 Roman Numerals (Continued)	X8 Reasoning -- Word Grouping	.296	13.80
	X5 Reasoning -- Number Relations	.321	18.93
	X6 Verbal Meaning -- Synonyms	.211	11.98
Y20 Total Topic: Numeration	X7 Spatial Relations -- Visualization	.117	4.34
	X8 Reasoning -- Word Grouping	.238	14.09
	X5 Reasoning -- Number Relations	.150	6.16
Y21 Concepts of Measurement	X6 Verbal Meaning -- Synonyms	.278	13.94
	X8 Reasoning -- Word Grouping	.224	10.92
	X3 Associative Memory	.140	5.53
Y22 Conversion of Units	X5 Reasoning -- Number Relations	.307	15.74
	X6 Verbal Meaning -- Synonyms	.283	14.09
	X1 Perceptual Discrimination -- Sets of Digits	.182	4.98
Y23 Operations - No Conversion	X2 Memory Span	-.130	1.04
	X5 Reasoning -- Number Relations	.250	10.11
	X6 Verbal Meaning -- Synonyms	.217	8.19

Table 18 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	% Contri- bution
Y24 Operations - With Conversion	X5 Reasoning -- Number Relations	.283	14.33
	X6 Verbal Meaning -- Synonyms	.249	12.78
	X8 Reasoning -- Word Grouping	.181	8.93
Y25 Total Topic: Measurement	X1 Perceptual Discrimination -- Sets of Digits	.119	3.60
	X5 Reasoning -- Number Relations	.286	16.71
	X6 Verbal Meaning -- Synonyms	.296	17.99
	X8 Reasoning -- Word Grouping	.214	12.55
Y26 Concepts	X2 Memory Span	-.124	1.06
	X5 Reasoning -- Number Relations	.155	6.99
	X6 Verbal Meaning -- Synonyms	.264	14.21
	X7 Spatial Relations -- Visualization	.125	4.23
	X8 Reasoning -- Word Grouping	.246	13.47
Y27 Addition	X3 Associative Memory	.120	4.17
	X5 Reasoning -- Number Relations	.183	8.27
	X8 Reasoning -- Word Grouping	.408	22.32

Table 18 (Continued)

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	% Contri- bution
Y28 Subtraction	X4 Associative Memory Plus Conversion	.126	4.84
	X5 Reasoning -- Number Relations	.200	9.06
	X6 Verbal Meaning -- Synonyms	.380	20.44
Y29 Multiplication	X4 Associative Memory Plus Conversion	.193	8.28
	X6 Verbal Meaning -- Synonyms	.267	14.43
	X7 Spatial Relations -- Visualization	.121	4.11
	X8 Reasoning -- Word Grouping	.234	12.46
Y30 Division	X4 Associative Memory Plus Conversion	.202	7.03
	X6 Verbal Meaning -- Synonyms	.331	13.91
Y31 Total Topic: Fractions	X4 Associative Memory Plus Conversion	.174	8.73
	X5 Reasoning -- Number Relations	.158	8.77
	X6 Verbal Meaning -- Synonyms	.331	22.03
	X7 Spatial Relations -- Visualization	.103	3.90
	X8 Reasoning -- Word Grouping	.223	14.11

^aSee section on Analyses for explanation of deleted statistics.

Research Objective 4
Relative Contributions by the Intellectual Processes:
Comparisons Among Mathematics Topics and Subtopics

The fourth research objective was to determine whether intellectual processes related to achievement differ among mathematics topics and subtopics. Tables 19 through 36 consist of data relevant to this objective. These data essentially are summaries of results pertinent to the first three objectives. The reader may refer to Tables 10 through 18 for specific statistics.

In addition to the summary tables which include the names of the contributing intellectual processes variables and the per cent contribution of each to the mathematics topics and subtopics, the present section contains a verbal description of the extant relationships. To facilitate this verbal description the names of the intellectual processes have been abbreviated as follows.

<u>Name of Intellectual Process</u>	<u>Abbreviation</u>
Perceptual Discrimination -- Sets of Digits	Perceptual discrimination
Memory Span	Memory span
Associative Memory	Associative memory
Associative Memory Plus Conversion	Associative memory -- conversion
Reasoning -- Number Relations	Reasoning -- numbers
Verbal Meaning -- Synonyms	Verbal meaning
Spatial Relations -- Visualization	Visualization
Reasoning -- Word Grouping	Reasoning -- words

The symbol (--) which appears in the tables indicates that the corresponding intellectual process did not make a significant contribution to the mathematics variable being considered.

Grade Four

Topic: Number, Operations, Assumptions

Presented in Table 19 is a summary of the results of the analyses for the topic: Number, Operations, and Assumptions.

Table 19

Contributing Processes Variables for Number, Operations, and Assumptions: Grade four

Intellectual Processes Variables	Mathematics Variable					
	Y1 Addi- tion	Y2 Subtrac- tion	Y3 Multi- plica- tion	Y4 Divi- sion	Y5 Laws and General- ization	Y6 Total Topic
X5 Reasoning -- Numbers	--	11.99 %	13.83 %	12.32 %	14.17 %	23.68 %
X8 Reasoning -- Words	--	--	--	--	--	--
X6 Verbal Meaning	--	8.26 %	10.21 %	--	11.26 %	13.12 %
X4 Associative Mem- ory--Conversion	--	--	--	--	--	--
X3 Associative Memory	3.60 %	3.44 %	--	--	--	2.79 %
X2 Memory Span	--	--	--	--	--	--
X1 Perceptual Discrimination	2.77 %	--	--	--	--	--
X7 Visualization	--	--	--	3.67 %	--	--

Reasoning-numbers accounted for a significant per cent of the variance in four of the five subtopics: Subtraction, Multiplication, Division, and Laws and generalizations. Verbal meaning appears next in order of prominence by its contribution to three subtopics: Subtraction, Multi-

plication, and Laws and generalizations. Associative memory and Perceptual discrimination, respectively, are the processes contributing to Addition. Associative memory also contributed to Subtraction. The processes which were predominantly common to the subtopics were also the processes contributing to the total Number, Operations, and Assumptions topic. These processes were Reasoning -- numbers, Verbal meaning, and Associative memory, respectively.

Topic: Geometry

Contained in Table 20 is a summary of the results of the analyses for the topic, Geometry.

Table 20

Contributing Processes Variables for Geometry: Grade four^a

Intellectual Processes Variables	Mathematics Variables			
	Y7 Definitions of Terms	Y8 Recognition of Geo. Fig.	Y9 Measurement of Geo. Fig.	Y10 Total Topic
X5 Reasoning -- Numbers	a	--	a	--
X8 Reasoning -- Words	a	--	a	--
X6 Verbal Meaning	a	12.18 %	a	9.02 %
X4 Associative Mem- ory -- Conversion	a	--	a	--
X3 Associative Memory	a	--	a	--
X2 Memory Span	a	--	a	.69 %
X1 Perceptual Discrimination	a	--	a	2.95 %
X7 Visualization	a	--	a	--

^aSee section on Analyses for explanation of deleted statistics.

Verbal meaning was the only one of the eight processes contributing to the subtopic, Recognition of geometric figures. Verbal meaning, Memory span, and Perceptual discrimination were the processes contributing to the total Geometry topic.

Topic: Relations

A summary of results of the analyses for the topic, Relations, is presented in Table 21.

Table 21
Contributing Processes Variables for Relations: Grade four^a

Intellectual Processes Variables	Mathematics Variables				
	Y11 Equality	Y12 Order Relation- ships	Y13 Maps, Graphs, Charts	Y14 Func- tions	Y15 Total Topic
X5 Reasoning -- Numbers	20.07 %	--	5.45 %	a	14.11 %
X8 Reasoning -- Words	--	--	--	a	--
X6 Verbal Meaning	12.36 %	10.95 %	7.69 %	a	22.89 %
X4 Associative Mem- ory--Conversion	--	--	--	a	--
X3 Associative Memory	--	--	--	a	3.37 %
X2 Memory Span	2.53 %	--	--	a	--
X1 Perceptual Discrimination	--	1.99 %	--	a	--
X7 Visualization	--	--	--	a	--

^aSee section on Analyses for explanation of deleted statistics.

Verbal meaning accounted for a significant per cent of the variance in the three subtopics analyzed: Equality, Order relationships, and Maps,

graphs, and charts. Next in order of prominence was Reasoning - numbers, which contributed to two of the three subtopics -- Equality and Maps, graphs, and charts. Memory span was a contributor to Equality, while Perceptual discrimination contributed to Order relationships. The two processes which were predominantly common to the subtopics were also processes contributing to the total Relations topic. These processes were Verbal meaning and Reasoning - numbers. In addition, Associative memory also contributed to the total topic.

Topic: Numeration

Table 22 contains a summary of the results of the analyses for the topic, Numeration.

Table 22

Contributing Processes Variables for Numeration: Grade Four

Intellectual Processes Variables	Mathematics Variables				
	Y16 Sets	Y17 Other Number Bases	Y18 Reading, Using Numbers	Y19 Roman Numerals	Y20 Total Topic
X5 Reasoning -- Numbers	22.04 %	--	31.46 %	11.58 %	15.43 %
X8 Reasoning -- Words	--	7.95 %	--	--	8.53 %
X6 Verbal Meaning	--	8.39 %	--	10.70 %	18.91 %
X4 Associative Mem- ory--Conversion	--	5.18 %	--	4.34 %	7.63 %
X3 Associative Memory	--	--	--	--	--
X2 Memory Span	--	--	--	1.97 %	--
X1 Perceptual Discrimination	--	3.16 %	--	--	4.49 %
X7 Visualization	--	--	--	--	--

An examination of this table reveals that Verbal meaning contributed to two subtopics : Other number bases, and Roman numerals, while Reasoning-numbers was a contributor to three of the four subtopics : Sets, Reading, writing, using numbers and Roman numerals. Associative memory-conversion accounted for a significant per cent of the variance in two subtopics : Other number bases, and Roman numerals. Two processes, Reasoning - words and Perceptual discrimination each contributed to Other number bases. Memory span contributed only to Roman numerals. Five processes contributed to the total Numeration topic. These five variables were also those predominantly common to the several subtopics; these processes were Reasoning--numbers, Reasoning--words, Verbal meaning, Associative memory - conversion, and Perceptual discrimination.

Topic: Measurement

Contained in Table 23 is a summary of the results of the analyses for the topic, Measurement.

Table 23

Contributing Processes Variables for Measurement: Grade Four

Intellectual Processes Variables	Mathematics Variables				
	Y21 Concepts of Mea- surement	Y22 Conver- sion of Units	Y23 Operations -No Con- version	Y24 Operations with Con- version	Y25 Total Topic
X5 Reasoning -- Numbers	8.51 %	16.05 %	12.49 %	8.66 %	21.63 %
X3 Reasoning -- Words	--	--	7.96 %	--	--
X6 Verbal Meaning	6.74 %	10.41 %	9.75 %	13.29 %	20.38 %
X4 Associative Mem- ory - Conversion	3.45 %	--	8.69 %	--	4.97 %
X3 Associative Memory	--	--	--	--	--
X2 Memory Span	2.62 %	--	2.14 %	1.80 %	2.32 %

Table 23 (continued)

Intellectual Processes Variables	Mathematics Variables				
	Y21 Concepts of Mea- surement	Y22 Conver- sion of Units	Y23 Operations -No Con- version	Y24 Operations with Con- version	Y25 Total Topic
X1 Perceptual Discrimination	--	--	--	--	--
X7 Visualization	--	--	--	--	--

Two processes, Reasoning--numbers and Verbal meaning, each contributed to all four of the subtopics within the Measurement topic. Memory span was a contributor to three of the four subtopics : Concepts of measurement, Operations - no conversion, and Operations - with conversion. Following Memory span in number of subtopics contributed to was Associative memory - conversion, which accounted for variance in two subtopics: Concepts of measurement and Operations - no conversion. Reasoning - words contributed to only one subtopic, Operations - no conversion. Four of the processes which were common to the subtopics were also the processes contributing to the total Measurement topic. These four processes were Reasoning - numbers, Verbal meaning, Associative memory - conversion, and Memory span.

Topic: Fractions

Presented in Table 24 is a summary of the results of the analyses for the topic, Fractions.

Table 24

Contributing Processes Variables for Fractions: Grade Four^a

Intellectual Processes Variables	Mathematics Variables					
	Y26 Concepts of Frac- tions	Y27 Addi- tion	Y28 Subtrac- tion	Y29 Multi- plica- tion	Y30 Divi- sion	Y31 Total Topic
X5 Reasoning -- Numbers	--	a	7.87 %	a	a	10.61 %
X8 Reasoning -- Words	--	a	--	a	a	--
X6 Verbal Meaning	9.70 %	a	13.26 %	a	a	20.23 %
X4 Associative Mem- ory - Conversion	--	a	10.12 %	a	a	6.33 %
X3 Associative Memory	2.42 %	a	--	a	a	3.34 %
X2 Memory Span	3.95 %	a	--	a	a	4.23 %
X1 Perceptual Discrimination	3.89 %	a	--	a	a	--
X7 Visualization	--	a	--	a	a	--

^aSee section on Analyses for explanation of deleted statistics.

Only one process was common to more than one subtopic within this topic. This process was Verbal meaning which contributed to Concepts of fractions and Subtraction. Three other variables contributed to Concepts of fractions and not to any other subtopic. These processes were Associative memory, Memory span, and Perceptual discrimination. Contributors to Subtraction in addition to Verbal meaning were Reasoning-numbers and Associative memory-conversion. Those processes contributing to the total Fractions topic were the following: Reasoning-numbers, Verbal meaning, Associative memory plus conversion, Associative memory, and Memory span.

Grade Five

Topic: Number, Operations, and Assumptions

Table 25 consists of a summary of the results of the analyses for the topic, Number; Operations, and Assumptions.

Table 25

**Contributing Processes Variables for Number, Operations,
and Assumptions: Grade Five**

Intellectual Processes Variables	Mathematics Variables					
	Y1 Addi- tion	Y2 Subtrac- tion	Y3 Multi- plica- tion	Y4 Divi- sion	Y5 Laws and general- ization	Y6 Total Topic
X5 Reasoning -- Numbers	--	15.73 %	19.63 %	19.51 %	8.87 %	24.83 %
X8 Reasoning -- Words	--	--	11.14 %	10.16 %	6.23 %	12.45 %
X6 Verbal Meaning	--	10.45 %	--	--	7.34 %	10.22 %
X4 Associative Mem- ory - Conversion	9.41 %	--	5.44 %	--	--	7.99 %
X3 Associative Memory	--	--	--	--	--	--
X2 Memory Span	--	--	--	--	--	--
X1 Perceptual Discrimination	--	--	--	--	--	--
X7 Visualization	--	--	--	--	--	--

The most frequent contributor to Number, Operations, and Assumptions was Reasoning -- numbers. This process contributed to four of the five sub-topics: Subtraction, Multiplication, Division, and Laws and generalizations. The next most prominent process was Reasoning -- words, by its

contributions to three subtopics: Multiplication, Division, and Laws and generalizations. Two processes, Verbal meaning and Associative memory - conversion each contributed to two subtopics. Verbal meaning contributed to Subtraction and Laws and generalizations, while Associative memory - conversion was a contributor to Addition and Multiplication. The processes contributing to the total Number, Operations, and Assumptions topic were the same four processes which accounted for portions of the variance in the several subtopics: Reasoning -- numbers, Reasoning -- words, Verbal meaning, and Associative memory - conversion.

Topic: Geometry

Table 26 contains a summary of the results of the analyses for the topic, Geometry.

Table 26

Contributing Processes Variables for Geometry: Grade Five^a

Intellectual Processes Variables	Mathematics Variables			
	Y7 Definitions of Terms	Y8 Recognition of Geo. Fig.	Y9 Measurement of Geo. Fig.	Y10 Total Topic
X5 Reasoning -- Numbers	a	12.55 %	15.23 %	16.89 %
X8 Reasoning -- Words	a	--	11.23 %	7.02 %
X6 Verbal Meaning	a	9.27 %	--	8.64 %
X4 Associative Mem- ory - Conversion	a	--	--	--
X3 Associative Memory	a	--	--	--
X2 Memory Span	a	.96 %	--	--
X1 Perceptual Discrimination	a	--	--	--
X7 Visualization	a	5.09 %	--	4.80 %

^aSee section on Analyses for explanation of deleted statistics.

Only one variable was a contributor to more than one subtopic within the Geometry topic. This process was Reasoning -- numbers which contributed to Recognition of geometric figures and Measurement of geometric figures. Reasoning -- words contributed to Measurement of geometric figures, while Verbal meaning, Memory span, and Visualization were contributors to Recognition of geometric figures. The processes which contributed to the total topic were Reasoning -- numbers, Reasoning -- words, Verbal meaning, and Visualization.

Topic: Relations

Contained in Table 27 is a summary of the results of the analyses for the topic, Relations.

Table 27

Contributing Processes Variables for Relations: Grade Five

Intellectual Processes Variables	Mathematics Variables				
	Y11 Equality	Y12 Order Relationships	Y13 Maps, Graphs, Charts	Y14 Func- tions	Y15 Total Topic
X5 Reasoning -- Numbers	16.74 %	20.55 %	--	14.90 %	26.21 %
X3 Reasoning -- Words	7.19 %	19.50 %	9.25 %	16.88 %	17.43 %
X6 Verbal Meaning	8.81 %	--	11.13 %	--	18.37 %
X4 Associative Mem- ory - Conversion	--	--	--	--	--
X3 Associative Memory	--	--	--	--	--
X2 Memory Span	--	--	--	1.79 %	--
X1 Perceptual Discrimination	--	--	--	--	--
X7 Visualization	--	--	--	--	--

Highest in frequency of contribution was Reasoning -- words which contributed to all four of the subtopics : Equality, Order relationships, Maps, graphs and charts, and Functions. Reasoning -- numbers contributed to three of the four subtopics : Equality, Order relationships, and Functions, while Verbal meaning contributed to two subtopics : Equality and Maps, graphs, and charts. Memory Span contributed only to Functions. An examination of total topic reveals that the processes predominantly common to the subtopics were also the processes contributing to the total Relations topic : Reasoning -- numbers, Reasoning -- words, and Verbal meaning.

Topic: Numeration

Table 28 contains a summary of the results of the analyses of the topic, Numeration.

Table 28

Contributing Processes Variables for Numeration: Grade Five

Intellectual Processes Variables	Mathematics Variables				
	Y16 Sets	Y17 Other Number Bases	Y18 Reading, Using Numbers	Y19 Roman Numerals	Y20 Total Topic
X5 Reasoning -- Numbers	--	20.07 %	15.11 %	16.72 %	24.04 %
X8 Reasoning -- Words	--	--	9.49 %	--	11.71 %
X6 Verbal Meaning	8.04 %	8.86 %	14.38 %	15.96 %	19.17 %
X4 Associative Mem- ory - Conversion	--	--	--	--	--
X3 Associative Memory	--	--	--	--	--
X2 Memory Span	--	--	--	--	--

Table 28 (continued)

Intellectual Processes Variables	Mathematics Variables				
	Y16 Sets	Y17 Other Number Bases	Y18 Reading, Using Numbers	Y19 Roman Numerals	Y20 Total Topic
X1 Perceptual Discrimination	--	--	2.81 %	--	--
X7 Visualization	4.49 %	--	--	--	--

One process, Verbal meaning, contributed to all four of the subtopics within this topic. Following Verbal meaning in frequency of contribution was Reasoning -- numbers which contributed to three of the four subtopics: Other number bases, Reading, writing, using numbers, and Roman numerals. Three processes each contributed to only one of the subtopics: Reasoning -- words was a contributor to Reading, writing, using numbers; Perceptual discrimination contributed to Reading, writing, using numbers; and Visualization was a contributor to Sets. The three processes contributing to the total Numeration topic were Reasoning -- numbers, Reasoning -- words, and Verbal meaning.

Topic: Measurement

Presented in Table 29 is a summary of the results of the analyses for the topic, Measurement.

Table 29

Contributing Processes Variables for Measurement: Grade Five

Intellectual Processes Variables	Mathematics Variables				
	Y21 Concepts of Mea- surement	Y22 Conver- sion of Units	Y23 Operations -No Con- version	Y24 Operations with Con- version	Y25 Total Topic
X5 Reasoning -- Numbers	8.22 %	17.20 %	10.72 %	25.37 %	24.63 %

Table 29 (continued)

Intellectual Processes Variables	Mathematics Variables				
	Y21 Concepts of Mea- surement	Y22 Conver- sion of Units	Y23 Operations -No Con- version	Y24 Operations with Con- version	Y25 Total Topic
X8 Reasoning -- Words	8.47 %	7.28 %	13.47 %	--	11.73 %
X6 Verbal Meaning	7.89 %	13.42 %	12.18 %	--	13.93 %
X4 Associative Mem- ory - Conversion	--	--	--	--	--
X3 Associative Memory	--	--	--	--	--
X2 Memory Span	--	--	--	--	--
X1 Perceptual Discrimination	--	--	--	--	--
X7 Visualization	5.33 %	--	--	6.89 %	8.20 %

Reasoning -- numbers contributed a significant per cent of the variance to all four of the subtopics within this topic : Concepts of measurement, Conversion of units, Operations - no conversion, Operations - with conversion. Two processes, Reasoning -- words and Verbal meaning each contributed to the same three subtopics : Concepts of measurement, Conversion of units, and Operations - no conversion. Visualization was a contributor to two subtopics : Concepts of measurement and Operations - with conversion. The same four processes which contributed to the several subtopics were also the processes contributing to the total Measurement topic : Reasoning -- numbers, Reasoning -- words, Verbal meaning, and Visualization.

Topic: Fractions

A summary of the results of the analyses for the topic, Fractions, is presented in Table 30.

Table 30

Contributing Processes Variables for Fractions: Grade Five^a

Intellectual Processes Variables	Mathematics Variables					
	Y26 Concepts of Frac- tions	Y27 Addi- tion	Y28 Subtrac- tion	Y29 Multi- plica- tion	Y30 Divi- sion	Y31 Total Topic
X5 Reasoning -- Numbers	18.45 %	23.89 %	23.57 %	a	a	30.79 %
X8 Reasoning -- Words	--	17.67 %	--	a	a	8.01 %
X6 Verbal Meaning	--	--	--	a	a	7.33 %
X4 Associative Mem- ory -- Conversion	--	--	--	a	a	4.12 %
X3 Associative Memory	--	--	--	a	a	--
X2 Memory Span	1.95 %	--	--	a	a	.60 %
X1 Perceptual Discrimination	--	--	--	a	a	--
X7 Visualization	--	--	--	a	a	6.42 %

^aSee section on Analyses for explanation of deleted statistics.

Of the three subtopics analyzed, Reasoning -- Numbers contributed to each : Concepts of fractions, Addition, and Subtraction. Reasoning -- words contributed to one of the three subtopics -- Addition. Memory span also contributed to one subtopic -- Concepts of fractions. The processes which contributed to the total topic were the following: Reasoning -- numbers, Reasoning -- words, Verbal meaning, Associative memory -- conversion, Memory span, and Visualization.

Grade Six

Topic: Number, Operations, and Assumptions

Contained in Table 31 is a summary of the results of the analyses for the topic, Number, Operations, and Assumptions.

Table 31

Contributing Processes Variables for Number, Operations, and Assumptions: Grade Six

Intellectual Processes Variables	Mathematics Variables					
	Y1 Addition	Y2 Subtraction	Y3 Multiplication	Y4 Division	Y5 Laws and generalization	Y6 Total Topic
X5 Reasoning -- Numbers	--	--	--	--	7.47 %	9.29 %
X8 Reasoning -- Words	--	--	8.27 %	--	--	7.09 %
X6 Verbal Meaning	--	14.93 %	8.18 %	9.95 %	4.78 %	11.04 %
X4 Associative Memory - Conversion	2.82 %	--	3.55 %	8.18 %	--	5.52 %
X3 Associative Memory	--	--	7.39 %	--	--	--
X2 Memory Span	--	--	10.09 %	--	--	--
X1 Perceptual Discrimination	--	3.83 %	--	6.66 %	--	5.03 %
X7 Visualization	--	--	--	--	--	--

An inspection of this table reveals that one process, Verbal meaning, was a contributor to four of the five subtopics within this topic: Subtraction, Multiplication, Division, and Laws and generalizations. Next in order of prominence was Associative memory - conversion which contributed to three subtopics: Addition, Multiplication, and Division.

Perceptual discrimination contributed to two subtopics: Subtraction and Division. Four processes each contributed to only one subtopic: Reasoning -- numbers contributed to Laws and generalizations; Reasoning -- words contributed to Multiplication; Associative memory was a contributor to Multiplication; and Memory span also contributed to Multiplication. Five processes contributed to the total Number, Operations, and Assumptions topic: Verbal meaning, Associative memory - conversion, Perceptual discrimination, Reasoning -- numbers, and Reasoning -- words.

Topic: Geometry

Contained in Table 32 is a summary of the results of the analyses for the topic, Geometry.

Table 32

Contributing Processes Variables for Geometry: Grade Six^a

Intellectual Processes Variables	Mathematics Variables			
	Y7 Definitions of Terms	Y8 Recognition of Geo. Fig.	Y9 Measurement of Geo. Fig.	Y10 Total Topic
X5 Reasoning -- Numbers	a	--	13.74 %	12.63 %
X8 Reasoning -- Words	a	7.04 %	10.56 %	10.08 %
X6 Verbal Meaning	a	19.26 %	9.40 %	15.49 %
X4 Associative Mem- ory - Conversion	a	--	5.03 %	6.59 %
X3 Associative Memory	a	--	--	--
X2 Memory Span	a	--	--	--
X1 Perceptual Discrimination	a	--	--	--
X7 Visualization	a	4.32 %	3.96 %	5.01 %

^aSee section on Analyses for explanation of deleted statistics.

Three processes, Reasoning -- words, Verbal meaning, and Visualization, were all contributors to the two subtopics analyzed: Recognition of geometric figures and Measurement of geometric figures. Reasoning -- numbers and Associative memory -- conversion contributed to one subtopic -- Measurement of geometric figures. The processes which contributed to the total Geometry topic were the same processes contributing to the several subtopics: Reasoning -- words, Verbal Meaning, Visualization, Reasoning -- numbers, and Associative memory -- conversion.

Topic: Relations

A summary of the results of the analyses for the topic, Relations, is presented in Table 33.

Table 33

Contributing Processes Variables for Relations: Grade Six

Intellectual Processes Variables	Mathematics Variables				
	Y11 Equality	Y12 Order Relation- ships	Y13 Maps, Graphs, Charts	Y14 Func- tions	Y15 Total Topic
X5 Reasoning -- Numbers	14.94 %	16.08 %	6.70 %	8.04 %	20.88 %
X8 Reasoning -- Words	8.12 %	9.18 %	--	17.57 %	13.37 %
X6 Verbal Meaning	--	14.91 %	7.37 %	--	14.24 %
X4 Associative Mem- ory -- Conversion	--	--	--	--	--
X3 Associative Memory	--	--	--	--	--
X2 Memory Span	--	--	--	--	--
X1 Perceptual Discrimination	--	--	--	5.02 %	--
X7 Visualization	3.56 %	--	--	--	4.21 %

One processes, Reasoning -- numbers contributed to four of the four topics analyzed: Equality, Order relationships, Maps, graphs, and charts, and Functions. Next in order of prominence was Reasoning -- words, which contributed to three of the four subtopics: Equality, Order relationships, and Functions. Verbal meaning was a contributor to two subtopics: Order relationships and Maps, graphs, and charts. Perceptual discrimination contributed to Functions while Visualization also contributed to only one subtopic -- Equality. Four processes contributed to the total Relations topic: Reasoning -- numbers, Reasoning -- words, Verbal meaning, and Visualization.

Topic: Numeration

Table 34 contains a summary of the results of the analyses for the topic, Numeration.

Table 34

Contributing Processes Variables for Numeration: Grade Six

Intellectual Processes Variables	Mathematics Variables				
	Y16 Sets	Y17 Other Number Bases	Y18 Reading, Using Numbers	Y19 Roman Numerals	Y20 Total Topic
X5 Reasoning -- Numbers	6.53 %	8.15 %	14.05 %	9.73 %	18.93 %
X8 Reasoning -- Words	--	8.45 %	13.45 %	13.80 %	14.09 %
X6 Verbal Meaning	--	--	13.64 %	--	11.98 %
X4 Associative Mem- ory -- Conversion	--	--	--	--	--
X3 Associative Memory	--	--	--	5.32 %	--
X2 Memory Span	--	--	--	--	--
X1 Perceptual Discrimination	--	--	--	--	--

Table 34 (continued)

Intellectual Processes Variables	Mathematics Variables				
	Y16 Sets	Y17 Other Number Bases	Y18 Reading, Using Numbers	Y19 Roman Numerals	Y20 Total Topic
X7 Visualization	--	--	--	--	4.34 %

Reasoning -- numbers contributed to all four subtopics within this topic:

Sets, Other number bases, Reading, writing, using numbers, and Roman numerals. Appearing next in order of frequency was Reasoning -- words which contributed to three of the four subtopics : Other number bases, Reading, writing, and using numbers, and Roman Numerals. Verbal meaning contributed to only one subtopic -- Reading, writing, and using numbers, while Associative memory also contributed to only one subtopic -- Roman numerals. The processes which contributed to the total Numeration topic were Reasoning -- numbers, Reasoning -- words, Verbal meaning, and Visualization.

Topic: Measurement

A summary of the results of the analyses for the topic, Measurement, is in Table 35.

Table 35

Contributing Processes Variables for Measurement: Grade Six

Intellectual Processes Variables	Mathematics Variables				
	Y21 Concepts of Mea- surement	Y22 Conver- sion of Units	Y23 Operations -No Con- version	Y24 Operations with Con- version	Y25 Total Topic
X5 Reasoning -- Numbers	6.16 %	15.74 %	10.11 %	14.33 %	16.71 %
X8 Reasoning -- Words	10.92 %	--	--	8.93 %	12.55 %

Table 35 (continued)

Intellectual Processes Variables	Mathematics Variables				
	Y21 Concepts of Mea- surement	Y22 Conver- sion of Units	Y23 Operations -No Con- version	Y24 Operations with Con- version	Y25 Total Topic
X6 Verbal Meaning	13.94 %	14.09 %	8.19 %	12.78 %	17.99 %
X4 Associative Mem- ory -- Conversion	--	--	--	--	--
X3 Associative Memory	--	5.53 %	--	--	--
X2 Memory Span	--	--	1.04 %	--	--
X1 Perceptual Discrimination	--	--	4.98 %	--	3.60 %
X7 Visualization	--	--	--	--	--

An inspection of the table reveals that two processes, Reasoning -- numbers and Verbal meaning contributed to all four of the subtopics analyzed: Concepts of measurement, Conversion of units, Operations -- no conversion, and Operations with conversion. Reasoning -- words contributed to two subtopics: Concepts of measurement and Operations with conversion. Three processes each contributed to only one subtopic: Associative memory contributed to Conversion of units; Memory span contributed to Operations -- no conversion; Perceptual discrimination contributed to Operations -- no conversion. For the total Measurement topic, four processes were contributors: Reasoning -- numbers, Reasoning -- words, Verbal meaning, and Perceptual discrimination.

Topic: Fractions

Table 36 contains a summary of the results of the analyses for the topic, Fractions.

Table 36

Contributing Processes Variables for Fractions: Grade Six

Intellectual Processes Variables	Mathematics Variables					
	Y26 Concepts of Frac- tions	Y27 Addi- tion	Y28 Subtrac- tion	Y29 Multi- plica- tion	Y30 Divi- sion	Y31 Total Topic
X5 Reasoning -- Numbers	6.99 %	8.27 %	9.06 %	--	--	8.77 %
X8 Reasoning -- Words	13.47 %	22.32 %	--	12.46 %	--	14.11 %
X6 Verbal Meaning	14.21 %	--	20.44 %	14.43 %	13.91 %	22.03 %
X4 Associative Mem- ory -- Conversion	--	--	4.84 %	8.28 %	7.03 %	8.73 %
X3 Associative Memory	--	4.17 %	--	--	--	--
X2 Memory Span	1.06 %	--	--	--	--	--
X1 Perceptual Discrimination	--	--	--	--	--	--
X7 Visualization	4.23 %	--	--	4.11 %	--	3.90 %

One process, Verbal meaning, contributed to four of the five subtopics within this topic: Concepts of fractions, Subtraction, Multiplication, and Division. Next in order of prominence were three processes, Reasoning -- numbers, Reasoning -- words, and Associative memory -- conversion, each of which contributed to three subtopics. Reasoning -- numbers contributed to Concepts of fractions, Addition, and Subtraction; Reasoning -- words contributed to Concepts of fractions, Addition, and Multiplication; Associative memory -- conversion was a contributor to Subtraction, Multiplication, and Division. Visualization contributed to two subtopics -- Concepts of fractions, and Multiplication, while Memory span and Associative memory each contributed to only one subtopic --

Concepts of fractions and Addition, respectively. Four processes contributed to the total Fractions topic: these processes were Verbal meaning, Reasoning -- words, Reasoning -- numbers, and Associative memory -- conversion, and Visualization.

CHAPTER IV

DISCUSSION, CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS

Introduction

The purpose of this study, as specified in Chapter I, was to examine relationships between certain intellectual processes and achievement of instructional objectives for selected mathematics topics and subtopics taught at grade levels four, five, and six. The procedures used to attain this purpose are described in Chapter II. Chapter III is concerned with results of the statistical analyses of the data. In the present chapter, these results are summarized and discussed and then conclusions, limitations, and implications are proposed.

Summary of Results and Discussion

Briefly, two trends may be discerned in the statistical data. The first of these trends is the consistency with which two intellectual processes, Reasoning -- number relations and Verbal meaning -- synonyms, were predominant contributors to mathematics topics and subtopics at all three grade levels. The second, more tentative, trend is a tendency toward differentiation among grade levels in the various types of intellectual processes, other than Reasoning -- number relations and Verbal meaning -- synonyms, which contributed to the mathematics subtopics analyzed. These trends are considered in more detail below.

The first trend apparent in the statistical data is the consistency with which two intellectual processes, Reasoning -- number relations and Verbal meaning -- synonyms, were predominant contributors to mathematics topics and subtopics analyzed at all three grade levels. The information in Table 37 illustrates this trend.

Of the 19 subtopics analyzed at grade four, Reasoning -- numbers contributed to 14, while Verbal meaning contributed to 15. Reasoning -- numbers contributed to five of the six topics, and Verbal meaning

Table 37

Summary of the Contributions of Reasoning -- Numbers and Verbal Meaning to the Mathematics Topics and Subtopics^a

Mathematics Subtopics and Topics	X5 Reasoning-Numbers			X6 Verbal Meaning		
	Grade Level			Grade Level		
	4	5	6	4	5	6
Y1 Addition	0	0	0	0	0	0
Y2 Subtraction	X	X	0	X	X	X
Y3 Multiplication	X	X	X	X	0	X
Y4 Division	X	X	0	0	0	X
Y5 Laws and Generalizations	X	X	X	X	X	X
Y7 Definitions of Terms	-	-	-	-	-	-
Y8 Recognition of Geo. Fig.	0	X	0	X	X	X
Y9 Measurement of Geo. Fig.	-	X	X	-	0	X
Y11 Equality	X	X	X	X	X	0
Y12 Order Relations	0	X	X	X	X	X
Y13 Maps, Graphs, Charts	X	0	X	X	X	X
Y14 Functions	-	X	X	-	0	0
Y16 Sets	X	0	X	0	X	0
Y17 Number Bases	0	X	X	X	X	0
Y18 Reading, Writing, Using No.	X	X	X	0	X	X
Y19 Roman Numerals	X	X	X	X	X	0
Y21 Concepts of Measurement	X	X	X	X	X	X
Y22 Conversion of Units	X	X	X	X	X	X
Y23 Operations-No Conversion	X	X	X	X	X	X
Y24 Operations-With Conversion	X	X	X	X	0	X
Y26 Concepts	0	X	X	X	0	X
Y27 Addition	-	X	X	-	0	0
Y28 Subtraction	X	X	X	X	0	X
Y29 Multiplication	-	-	0	-	-	X
Y30 Division	-	-	0	-	-	X
Y6 Total Topic: No., Oper., Assump.	X	X	X	X	X	X
Y10 Total Topic: Geometry	-	X	X	-	X	X
Y15 Total Topic: Relations	X	X	X	X	X	X
Y20 Total Topic: Numeration	X	X	X	X	X	X
Y25 Total Topic: Measurement	X	X	X	X	X	X
Y31 Total Topic: Fractions	-	X	X	-	X	X

^aThe following code is used for the symbols in the Table: X indicates that the intellectual process contributed significantly to the aspect of mathematics being considered; 0 indicates that the intellectual process did not contribute significantly; - (one dash) indicates that analyses were not performed for the reasons listed in the Analyses section of Chapter II.

contributed to all six topics¹. At the fifth grade level, Reasoning -- numbers contributed to 19 of the 22 subtopics analyzed; Verbal meaning contributed to 13 subtopics. Both of these processes contributed to all six topics². The same general pattern is observed at the sixth grade level where Reasoning -- numbers contributed to 18 of the 24 subtopics analyzed and Verbal meaning contributed to 17 subtopics. Reasoning -- numbers and Verbal meaning each contributed to all six topics.

The evidence reported immediately above about the influence of Reasoning -- numbers and Verbal meaning -- synonyms generally is consistent with results obtained in ten of the twelve previous investigations containing data pertinent to the role of intellectual processes in various aspects of mathematics achievement. These ten investigators were Barrett (1952), Blackwell (1940), Corle (1958), Chase (1958), Holzinger and Swineford (1946), Murray (1949), Olckers (1951), Shine (1960), Stake (1958), and Sutherland (1942). In addition, the results cited above are partially consistent with data reported by the remaining two of the twelve investigators: namely, Lee (1955) and Hansen (1943).

In each of the twelve investigations batteries of intellectual processes tests were used; the number of tests in these batteries ranged from two to fifty-three. The tests were designed to measure such cognitive characteristics as memory, spatial ability, reasoning, vocabulary, and discrimination. For the sake of brevity, the research summary presented below includes only citation of the researchers' results relevant to the role of reasoning and verbal facility in mathematics achievement.

The work of the investigators who found that reasoning and verbal facility were important contributors to mathematics achievement is summarized below in terms of aspects of mathematics they studied: problem solving; algebra, geometry, and trigonometry; and total per-

^{1,2}The Geometry and Fractions topic scores at the fourth grade level and the Fractions topic scores at the fifth grade level alluded to in the text were attenuated: scores for certain subtopics not taught at these grade levels were not included in the topic scores. See the Analyses section of Chapter II for more details.

formance on batteries of mathematics achievement tests.

Corle (1958), Chase (1958), Olckers (1951) and Sutherland (1942) concentrated on problem solving in mathematics. Corle (1958) and Chase (1958) both found that verbal fluency and reasoning were prominent contributors to the problem solving performance of sixth grade pupils. Sutherland (1942) and Olckers (1951) obtained similar results with fifth and seventh grade pupils, respectively.

Barrett (1952), Blackwell (1940), Holzinger and Swineford (1946), and Murray (1949) were concerned with mathematics achievement of students at the junior and senior high school and college levels. They reported that reasoning and verbal meaning were the primary contributors to geometry at the junior high school level (Blackwell, 1940; Murray, 1949), at the senior high school level (Holzinger and Swineford, 1946), and at the college freshman level (Barrett, 1952). In addition to their results pertinent to geometry, Blackwell (1940) found reasoning and verbal meaning to be important contributors to algebra at the junior high school level while Barrett (1952) observed these variables to be prominent contributors to algebra and trigonometry at the freshman level of college.

Among the data reported by Stake (1958) and Shine (1960) was evidence germane to relationships between various cognitive characteristics and scores on the Stanford Achievement Test (Stake) and the Iowa Test of Basic Skills (Shine). In both studies, the data indicated that scores reflecting reasoning and verbal facility were significantly correlated with scores on the mathematics achievement tests.

As stated previously, two investigators obtained results which are only partially consistent with the results of the present study. Lee (1955) and Hansen (1943) found reasoning to be correlated with mathematics achievement; on the other hand, both investigators reported that their measures of verbal meaning and vocabulary did not correlate significantly with mathematics achievement. Hansen (1943) used tests of problem solving in mathematics while Lee used a battery of 53 instruments labeled attainment tests in mathematics.

In summary, the observation that Reasoning -- number relations and Verbal meaning -- synonyms are prominent contributors to the majority of aspects of mathematics examined in the present study

is consistent, in most cases, with the results of previous investigations involving other populations and other aspects of mathematics. Similarly, the predominance of the Reasoning -- number relations and Verbal meaning -- synonyms processes is understandable when the nature of these two variables is considered in the context of the way mathematics is taught in the schools at the present time¹.

Reasoning -- number relations refers to the associated abilities involved in finding general rules that will fit sets of data -- the forming and trying-out of hypotheses about relationships among numbers. Verbal meaning -- synonyms refers to the ability to understand ideas when they are expressed in words.

In recent years, major shifts in mathematics education have taken place. One shift has been in the nature of the mathematics taught. A major goal of contemporary mathematics programs has been to increase the pupils' knowledge about the ideas of mathematics and the applications of those ideas to various situations; attaining this goal requires that the pupils' understand mathematical concepts, principles, and vocabulary as well as the structure of mathematics. In view of these requirements related to the content of the mathematics to be learned it might be expected that the ability to understand or grasp relationships, to discover principles, and to generalize would be quite influential in achievement. In brief, achievement in mathematics, as it is taught currently, requires the pupil to reason (Brownell, 1945; Hartung, 1962).

The nature of modern mathematics also is highly verbal. Performance involved in learning the meaning of new terms, stating relationships, and generalizing is not confined to reproducing on paper a few mathematical symbols as evidence that a pupil comprehends such things; rather, there is a constant dialogue between pupil and teacher to insure that meaning and understanding are established (Van Engen, 1953). This more highly verbal approach to mathematics is reflected in the increased

¹Chapter II contains evidence that the mathematics curriculum and methodology to which the subjects in the present study were exposed have resulted from intensive local and state-wide efforts to develop a mathematics instructional program consistent with recommended trends and practices in contemporary mathematics education.

amount of space devoted to expository material in recently published series of mathematics text-books (e.g., School Mathematics Study Group, 1962).

Another shift in mathematics education has been in the nature of the methodology. Generally, there is no longer the traditional dichotomy of computation and "word problems", and also there is no longer the same emphasis upon rote memorization of facts and rules, per se. Efforts have been made to place increasing emphasis upon understanding the "why" of mathematics. Clark and Fehr (1953, p.341) made the following comments concerning this shift in emphasis.

The older practice of emphasizing skill learning with little emphasis upon meaning led to the justifiable criticism that instruction was mechanical, that the pupils learned the how without the why. Fortunately, the influence of the field psychology, with its emphasis upon concepts and relationship, is increasing.

The teaching approach used in contemporary mathematics programs has been, in many instances, the "discovery" method. With this method much stress is given to the process of formulating laws and generalizations which are applicable to the particular aspect of mathematics being studied. In essence, the "discovery" method usually consists of a rational sequential analyzing and ordering of information concluding with the formulation of a principle or generalization. On an a priori basis alone one might expect that the primary intellectual process which would be required for this type of activity is basically one of reasoning (Brownell and Hendricksen, 1950; Brownell and Sims, 1946; Flournoy, 1957; Gibb, 1956; Hendrix, 1950). With the increasing emphasis upon the use of a rational approach to understanding and learning mathematics, there has been a tendency to relegate rote memorizing of facts to a lower order of emphasis in the hierarchy of modern mathematics teaching methods. That is, while memorization of factual material continues to be recognized as an important and necessary activity in the successful mastery of certain aspects of mathematics, the tendency has been to de-emphasize it as a major process to be used in the learning of mathematics. This tendency is reflected in many of the new series of mathematics textbooks and teachers' manuals at the elementary school level (e.g., Eicholz, O'Daffer, Brumfiel, and Shanks, 1964). In these materials, space

and instructions for practice or "drill" have been considerably reduced; and concomitantly, there has been a corresponding increase in the instructions and the space devoted to questions and activities designed to elicit from pupils responses which ostensibly would seem to involve their abstracting or deriving principles, applying these principles to specific cases, and generalizing from these situations to make further applications of these derived principles.

The second, more tentative, trend apparent in the statistical data obtained in the present study is the tendency toward differentiation among grade levels in the various types of intellectual processes, other than Reasoning -- number relations and Verbal meaning -- synonyms, which contributed to the mathematics subtopics¹ analyzed. The information in Table 38 illustrates this tentative trend.

After Reasoning -- numbers and Verbal meaning -- synonyms were eliminated from consideration, grade four and grade five tended to have different patterns of processes contributing to subtopic achievement, while grade six tended to have a pattern which overlapped with both grade four and grade five to some extent. Generally, Perceptual discrimination, Memory span, Associative memory, and Associative memory plus conversion were frequent contributors to mathematics subtopics at grade four and, with the exception of Memory span, at grade six while these processes contributed to relatively fewer subtopics at grade five. On the other hand, grade six also was similar to grade five with respect to the frequency with which Reasoning -- word grouping and Visualization contributed to the subtopics.

An intensive search of the literature revealed no previous investigations which encompassed within a single study comparison of intellectual processes contributing to different groups' achievement in a variety of specific mathematics subtopics. Most investigators restricted their work to one sample; also, they studied total mathematics achievement test scores or delimited one aspect of mathematics

¹It was not possible to determine in the present study whether a similar differentiation among processes contributing to topic scores would occur. This is so because of the small number of topic scores involved and because several of the topic scores were not comparable among grade levels as a result of the necessity to delete certain subtopic scores from the analyses (Analyses section, Chapter II).

Table 38
Summary of Contributions of Intellectual Processes, Other than
Reasoning -- Numbers and Verbal Meaning, to the Mathematics Subtopics

Mathematics Subtopics	X1 Perceptual Discrimination						X2 Memory Span			X3 Associative Memory		
	Grade Level						Grade Level			Grade Level		
	4	5	6	6	6	6	4	5	6	4	5	6
Y1 Addition	X	0	0	0	0	0	0	0	0	X	0	0
Y2 Subtraction	0	0	X	X	0	0	0	0	0	X	0	0
Y3 Multiplication	0	0	0	X	0	0	0	0	0	0	0	X
Y4 Division	0	0	0	X	0	0	0	0	0	0	0	0
Y5 Laws and Generalizations	0	0	0	0	0	0	0	0	0	0	0	0
Y7 Definitions of Terms	-	-	-	-	-	-	-	-	-	-	-	-
Y8 Recognition of Geometric Figures	0	0	0	0	0	0	0	X	0	0	0	0
Y9 Measurement of Geometric Figures	-	0	0	0	0	0	-	0	0	-	0	0
Y11 Equality	0	0	0	0	0	0	X	0	0	0	0	0
Y12 Order Relations	X	0	0	0	0	0	0	0	0	0	0	0
Y13 Maps, Graphs, Charts	0	0	0	0	0	0	0	0	0	0	0	0
Y14 Functions	-	0	0	X	0	0	-	X	0	-	0	0
Y16 Sets	0	0	0	0	0	0	0	0	0	0	0	0
Y17 Number Bases	X	0	0	0	0	0	0	0	0	0	0	0
Y18 Reading, Writing, Using Numbers	0	X	0	0	0	0	0	0	0	0	0	0
Y19 Roman Numerals	0	0	0	0	0	0	X	0	0	0	0	X
Y21 Concepts of Measurement	0	0	0	0	0	0	X	0	0	0	0	0
Y22 Conversion of Units	0	0	0	0	0	0	0	0	0	0	0	X
Y23 Operations-No Conversion	0	0	X	X	0	X	X	0	0	0	0	0
Y24 Operations-With Conversion	0	0	0	0	0	0	X	0	0	0	0	0
Y26 Concepts	X	0	0	0	0	0	X	X	X	X	0	0
Y27 Addition	-	0	0	0	0	0	-	0	0	-	0	X
Y28 Subtraction	0	0	0	0	0	0	0	0	0	0	0	0
Y29 Multiplication	-	-	0	0	0	0	-	-	0	-	-	0
Y30 Division	-	-	0	0	0	0	-	-	0	-	-	0

Table 38 (continued)

Mathematics Subtopics	X4						X7						X8					
	Associative Memory Plus Conversion						Visualization						Reasoning - Word Grouping					
	4	5	6	4	5	6	4	5	6	4	5	6	4	5	6	4	5	6
Y1 Addition	0	X	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y2 Subtraction	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y3 Multiplication	0	X	0	0	0	0	0	0	0	0	0	0	0	X	X	0	X	X
Y4 Division	0	0	X	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y5 Laws and Generalizations	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y7 Definitions of Terms	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Y8 Recognition of Geometric Figures	0	0	0	0	0	0	0	X	X	0	0	0	0	0	0	0	0	X
Y9 Measurement of Geometric Figures	-	0	X	-	0	X	-	0	X	-	0	X	-	X	X	-	X	X
Y11 Equality	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y12 Order Relations	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y13 Maps, Graphs, Charts	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y14 Functions	-	0	0	-	0	0	-	0	0	-	0	0	-	0	0	-	0	0
Y16 Sets	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y17 Number Bases	X	0	0	0	0	0	0	0	0	0	0	0	X	0	0	0	0	0
Y18 Reading, Writing, Using Numbers	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y19 Roman Numerals	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y21 Concepts of Measurement	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y22 Conversion of Units	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y23 Operations-No Conversion	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y24 Operations-With Conversion	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y26 Concepts	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y27 Addition	-	0	0	-	0	0	-	0	0	-	0	0	-	0	0	-	0	0
Y28 Subtraction	X	0	X	0	0	X	0	0	0	0	0	0	0	0	0	0	0	0
Y29 Multiplication	-	-	X	-	-	X	-	-	-	-	-	X	-	-	-	-	-	-
Y30 Division	-	-	X	-	-	X	-	-	-	-	-	0	-	-	-	-	-	0

^aThe following code is used for the symbols in the Table: X indicates that the intellectual processes contributed significantly to the aspect of mathematics being considered; 0 indicates that the intellectual processes did not contribute significantly; - (one dash) indicates that analyses were not performed for the reasons listed in the Analyses section of Chapter II.

for study, e.g., problem solving or geometry. Therefore, the results of the present study indicating a tendency toward differentiation among grade levels are unique.

Tentatively, it may be hypothesized that the differentiation in intellectual processes, other than Reasoning -- numbers and Verbal meaning, contributing to the subtopics at grade levels four, five, and six may be a function of differential curriculum emphases at these grade levels. This tentative hypothesis is discussed below in terms of the definitions of the intellectual processes involved and the curricular emphases in contemporary mathematics programs¹ for the intermediate grades.

Perceptual discrimination includes speed in finding figures, making comparisons, and carrying out other very simple tasks involving perception. Memory span refers to the ability to recall perfectly for immediate reproduction a list or series of items after only one presentation of the series. Associative memory refers to the ability to form and remember arbitrary associations between discrete stimuli and responses. Associative memory plus conversion refers to the ability to form and remember arbitrary associations between discrete stimuli and responses and to solve problems involving these new associations. Reasoning -- word grouping refers to the ability to identify concepts: to group words in terms of common attributes. Visualization refers to the ability to visualize objects and figures rotated in space and the relation between them.

A review of current mathematics education programs indicates that there is a tendency to introduce more new aspects of mathematics at the fourth and sixth grade levels than at the fifth grade level. At the fourth and sixth grade levels the learning of terms, concepts, and principles being introduced for the first time might be expected to require more use of intellectual processes such as memory, discrimination, etc. as well as reasoning and verbal facility. At the fifth

¹Chapter II contains evidence that the curriculum and methodology to which the subjects in the present study were exposed has resulted from intensive local and state-wide efforts to develop a mathematics instructional program consistent with recommended trends and practices in contemporary mathematics education.

grade level, it appears that there is a curricular concentration on exploring in depth concepts and principles which have been previously introduced. This exploration in depth might be expected to require more use of verbal conceptualization and visualization as well as reasoning with number quantities and facility in determining verbal meaning. In addition to learning new materials, new terms, and new concepts, pupils at the sixth grade level also continue to explore in depth those aspects of mathematics which have been introduced previously. The sixth grade is similar to the fifth grade in the latter respect. Thus, it would follow that conceptualization and visualization probably would be required for exploring in depth certain aspects of mathematics appearing at the two respective grade levels. In summary, the fourth and sixth grades show similar patterns of certain intellectual processes (perceptual discrimination, associative memory, and associative memory plus conversion) contributing to mathematics subtopics while the fifth and sixth grades are similar with respect to the contribution of the conceptualization and visualization processes. This apparent trend of differentiation in contributing processes among grade levels is not unexpected when the differential curriculum emphases at these grade levels are examined.

Conclusions

The results obtained in the present study are not inconsistent with the results of previous research involving different populations and diverse aspects of mathematics. Also, these results are understandable when they are considered within the context of the nature of the mathematics content which is taught and the methods and sequences by which this mathematics content is presented in contemporary mathematics programs represented by the program to which the subjects in the present study were exposed.

On the basis of the trends apparent in the obtained data and considerations related to results of previous research and to practices in contemporary mathematics education, the following conclusions would appear to be warranted.

1. Reasoning -- number relations and Verbal meaning -- synonyms are intellectual processes which are related predominantly

to achievement of instructional objectives for subtopics of the following mathematics topics as they are taught at grade levels four, five, and six: Number, operations and assumptions; Geometry; Numeration; Relations; Measurement; Fractions.

2. A tendency toward differentiation occurs among grade levels in intellectual processes, other than Reasoning -- number relations and Verbal meaning -- synonyms, which are related to achievement of instructional objectives for mathematics subtopics taught at grades four, five, and six. Perceptual discrimination, Memory span, Associative memory and Associative memory plus conversion are frequently related to achievement in mathematics subtopics at grade four and, with the exception of Memory span, at grade six while these processes contribute to relatively fewer subtopics at grade five. On the other hand, Visualization and Reasoning -- word grouping are more frequently related to subtopics at grades five and six but not at grade four.

Limitations

Inherent in the procedures and samples used in this study are six limitations to internal and external validity (Campbell and Stanley, 1963) which merit consideration. Extension of conclusions beyond these limitations will require further investigation.

First, the study was restricted to intellectual processes which might contribute to variation in mathematics achievement. Other influences also might contribute to variation in mathematics achievement: e.g., situational influences such as differences among teachers; emotional and motivational influences such as pupils' anxieties and interests specific to mathematics; physical influences such as pupils' sensory adequacy; and so on. Efforts were made to prevent any systematic biases related to these types of influences by such procedures as using restricting criteria in selecting subjects, etc. Theoretically, any of these influences which were operating were represented in the portions of the variance which were unexplained. Further research will be necessary to determine to what extent other influences such as those cited here are related to mathematics achievement.

Second, not all aspects of mathematics achievement nor all types of intellectual processes were sampled. The domains of relevant behavior are extensive and time and funds were restricted. Further research will be required to study behavior not assessed in the present study.

Third, generalizations which can be made on the basis of the obtained

data are limited, to some extent, to instructional programs of the type used to teach the pupils mathematics and to populations of pupils similar to those at grade levels four, five, and six in the Clarke County (Georgia) School District. Since mathematics represents specifically learned behavior, such limitations would seem to apply to any single study of mathematics achievement which is not extremely extensive in scope -- more extensive than the scope possible within the budgetary and time restrictions of the present study. The mathematics instructional program in Clarke County has a number of positive features and for several years has been the subject of intensive efforts for improvement. Also, efforts have been made to attain consistency among classes in procedures related to the several elements of the mathematics program such as scope, sequence, timing, methods, materials, allocation of personnel and facilities, and organization of pupils for instruction. In addition, none of the available evidence indicated that the pupils employed in the samples represented a markedly atypical population. However, generalizations are limited to populations and instructional programs similar to the ones described herein. Further work will be necessary to extend these generalizations to different populations which have different backgrounds of mathematics training.

Fourth, because of a rather limited amount of testing time in the schools, it was necessary to restrict the number of test items which could be used for assessing achievement in the mathematics subtopics. As a result, the reliability coefficients for the majority of the subtopic scores were relatively low. However, they were considered sufficiently high to yield for research purposes some preliminary information about trends in group behavior. Nevertheless, in order to obtain less equivocal results, it will be necessary to replicate the study using subtopic tests of greater length and higher reliability.

Fifth, because of time and budgetary restrictions as well as other practical considerations, the focus of the study was limited to the content of the mathematics instructional program at grade levels four, five, and six. Similarly important is an examination of such problems as the following: the nature of the relationships proposed for study herein at other grade levels, developmental trends in such relationships across grade levels from the primary school through the high

school, and so on. Such extensions will require further investigations which encompass the content of the instructional program at other grade levels.

Sixth, the procedures involved sampling mathematics achievement and intellectual processes after subjects completed the instructional program at the given grade level and essentially before they began the instructional program at a subsequent grade level. This timing yielded information pertinent to one facet of the problem delimited for study. Another facet of the problem is also important: namely, relationships among intellectual processes and achievement assessed during or immediately after the period when the pupils are participating in instruction in the particular mathematics content being assessed. This latter facet will require a subsequent investigation which is much more extensive than it was possible to conduct within the time and budgetary resources available for the present study.

Implications

As stated in Chapter I, the basic underlying interest in the present study was primarily heuristic: viz., to obtain information about psychological components of mathematics achievement in order to specify a relational bridge between two domains -- (1) mathematics education and (2) psychological research on variables affecting learning and on the nature of intelligence. This endeavor was considered educationally significant in the sense that identifying psychological components of mathematics achievement could be a useful preparatory step in the process of using psychological research as a partial basis for decision making about specific mathematics education procedures.

Following Underwood et.al. (1958, p. 8), it was expected that achievement of instructional objectives for the mathematics topics and subtopics selected for study herein "...might be shown to be a certain part rote learning, a certain part concept formation, and a certain part problem solving". As it happened, the data indicated that this was the case. The predominant psychological components were facility in numerical reasoning and facility in discerning verbal meaning while facility in verbal conceptualization, visualization, associative memory, associative memory plus conversion, memory span, and perceptual dis-

crimination were more minor components.

The results of the present study can be viewed with some confidence for at least two reasons: these results are not inconsistent with data obtained by previous investigators who employed samples from other populations and studied diverse aspects of mathematics; these results are understandable in the context of the content and methodology of contemporary mathematics education. Nevertheless, because of the limitations to internal and external validity cited immediately above in the section on Limitations, the results reported herein need to be verified and extended. Should these results remain consistent, the next task is to derive specific implications of pertinent psychological research which can be used as a partial basis for verifying or revising current pedagogical practices or for devising new pedagogical practices. That is, the research literature is replete with information about variables and conditions which affect childrens' reasoning, verbalization, conceptualization, memory, discrimination, etc. Similarly, more information is being obtained about these topics currently. A next step is a series of applied research projects and development projects designed to derive and field test the implications of such psychological data and other pertinent considerations for variations in the scope, sequence, timing, methods, and materials for a mathematics instructional program encompassing mathematics topics and subtopics at grades four, five and six like those which were the focus of the present study.

CHAPTER V

SUMMARY

Problem

Purpose and Objectives

The purpose of this study was to examine relationships between certain intellectual processes and achievement of instructional objectives for selected mathematics topics and subtopics taught at grade levels four, five, and six. The collection of evidence relevant to this purpose was guided by four specific research objectives.

1. To identify intellectual processes which are related, singly or in combination, to achievement of instructional objectives for each mathematics topic and subtopic.
2. To determine to what extent the identified intellectual processes or combinations of intellectual processes are related to achievement of instructional objectives for each mathematics topic and subtopic.
3. To specify the relative contribution, or weighting, of each intellectual process when combinations of intellectual processes are related to achievement of instructional objectives for each mathematics topic and subtopic.
4. To determine whether intellectual processes related to achievement differ among mathematics topics and subtopics.

Educational Significance

Essentially, the interest in this study was to obtain information which could be helpful in specifying a relational bridge between two domains: (1) mathematics education and (2) psychological research on variables affecting learning and on the nature of intelligence. Information obtained in a study like this could be useful to those responsible for planning and conducting applied research projects and development projects related to various educational problems.

Related Research

Research considered relevant to this study was restricted to empirical investigations of how various intellectual processes are related to mathematics achievement. Twelve investigations contained data which are pertinent, to some extent, to the purpose of the present study. Among aspects of mathematics considered in these investigations were topics in arithmetic, algebra, geometry, and trigonometry. In the various studies, subjects were selected from all school levels -- elementary, high school, and college. Intellectual processes studied included variables like perception of spatial relationships, reasoning, verbal comprehension, word fluency, and memory.

Procedure

Research Design

The basic research design was a response-response type of design. One set of responses was the subjects' scores on instruments chosen to assess achievement of selected mathematics instructional objectives. The other set of responses was the subjects' scores on instruments chosen to assess selected intellectual processes. The design was replicated for each subject. Briefly, the procedures used were as follows. Classes of fourth, fifth, and sixth grade pupils were administered mathematics achievement tests and intellectual processes tests. The tests were administered by classroom teachers.

Subjects

Eligible for inclusion in the samples were those pupils in the Clarke County (Georgia) School District who were taught in the mathematics instructional program at grade levels four, five, and six during the 1963-64 academic year and who satisfied other selection criteria. Pupils were selected at each grade level during the spring of 1964. The total group included 765 subjects: 244 in the fourth grade sample, 261 in the fifth grade sample, and 260 in the sixth grade sample.

Data and Instrumentation

The mathematics data were interval scores on tests designed to assess achievement of instructional objectives for each of the following selected topics and subtopics.

1. Number, Operations, and Assumptions: Addition, Subtraction, Multiplication, Division, and Laws and generalizations.
2. Geometry: Definitions of terms, Recognition of geometric figures, and Measurement of geometric figures.
3. Relations: Equality, Order relations, Maps, graphs, charts, and Functions.
4. Numeration: Sets, Other number bases, Reading, writing, using numbers, and Roman numerals.
5. Measurement: Concepts, Conversion of units, Operations with no conversion, and Operations with conversion.
6. Fundamental Operations with Fractions: Concepts, Addition, Subtraction, Multiplication, and Division.

The intellectual processes data were interval scores on tests designed to assess the following variables: Perceptual discrimination -- sets of digits, Memory span, Associative memory, Associative memory plus conversion, Reasoning -- number relations, Verbal meaning -- synonyms, Spatial relations -- visualization, and Reasoning -- word grouping.

General Procedures

Major steps involved in data collection included the following activities. Classes of pupils were selected from the total number available on the basis of criteria related to the teachers' training and experience; then, pupils who were eligible in terms of the remaining subject selection criteria were identified. Classroom teachers administered the instruments. Prior to the administration of the tests, a detailed testing schedule was formulated with the cooperation of appropriate school personnel. The testing sessions were scheduled so that all selected classes were administered the tests at approximately the same time. Meetings were held with teachers and principals in

order to orient them to the project. Following the orientation sessions, conferences were arranged for individual teachers who wished additional assistance in preparing for the test administration.

Test materials for pupils were prepared, packaged, and delivered to the schools prior to each test administration session. The instruments selected to sample the intellectual processes and mathematics achievement were administered to all pupils in the specified classes. All testing materials were collected from the respective schools immediately following each test administration session.

Analyses

The data consisted of interval scores obtained on the mathematics achievement tests and the intellectual processes tests.

Comparisons were made of the fourth, fifth, and sixth grade pupils' mean performance on the mathematics topics and subtopics and on the intellectual processes tests. The performance of each group was described in terms of means and standard deviations. A single classification analysis of variance was used to analyze relationships among the groups' means. When F-ratios were significant, the following techniques were used for making paired comparisons: t-tests of differences between means and F-tests of homogeneity of within-group variances.

Step-wise regression analyses were employed to obtain statistical data pertinent to the four research objectives. These analyses were performed separately for the mathematics topics and subtopics examined at each grade level.

The .05 probability level was the criterion for rejecting all null hypotheses except those pertaining to homogeneity of variance; for these latter tests, the .001 probability level was the criterion for rejecting the null hypotheses.

Results

The results of the analyses of the fourth, fifth, and sixth grade groups' relative performance on the mathematics topics and subtopics examined and on the intellectual processes tests indicated the follow-

ing. Although there were exceptions, the differences between the groups' means were in the expected direction: the fifth grade and sixth grade groups' means exceeded the fourth grade group's means while the fifth grade group's means exceeded those of the fourth grade group.

Two trends could be discerned in the statistical data yielded by the step-wise regression analyses. The first of these trends was the consistency with which two intellectual processes, Reasoning -- number relations and Verbal meaning -- synonyms, were predominant contributors to mathematics topics and subtopics at all three grade levels. The second, more tentative, trend was the tendency toward differentiation among grade levels in the various types of intellectual processes, other than Reasoning -- number relations and Verbal meaning -- synonyms, which contributed to the mathematics subtopics analyzed. After Reasoning -- numbers and Verbal meaning -- synonyms were eliminated from consideration, grade four and grade five tended to have different patterns of processes contributing to subtopic achievement, while grade six tended to have a pattern which overlapped with those of both grade four and grade five to some extent. Generally, Perceptual discrimination, Memory span, Associative memory, Associative memory plus conversion were frequent contributors to mathematics subtopics at grade four and, with the exception of Memory span, at grade six while these processes contributed to relatively fewer subtopics at grade five. On the other hand, grade six also was similar to grade five with respect to the frequency with which Reasoning -- word grouping and Visualization contributed to the subtopic achievement.

Discussion, Conclusions, Limitations, and Implications

Discussion

The results obtained in the present study are not inconsistent with the results of previous research involving different populations and diverse aspects of mathematics. In addition, these results are understandable when they are considered within the context of the nature of the mathematics content which is taught and the methods and sequences by which this mathematics content is presented in contemporary

mathematics education programs represented by the instructional program to which the subjects in the present study were exposed.

Conclusions

Concomitant weighing of the trends apparent in the obtained data of the consistency of these data with results reported by others, and of the reasonableness of these data in the context of contemporary practices in mathematics education provided a basis for considering two conclusions to be tenable.

1. Reasoning -- number relations and Verbal meaning -- synonyms are intellectual processes which are related predominantly to achievement of instructional objectives for subtopics of the following mathematics topics as they are taught at grade levels four, five, and six: Number, operations, and assumptions; Geometry; Numeration; Relations; Measurement; Fractions.
2. A tendency toward differentiation occurs among grade levels in intellectual processes, other than Reasoning -- number relations and Verbal meaning -- synonyms, which are related to achievement of instructional objectives for mathematics subtopics taught at grades four, five, and six. Perceptual discrimination, Memory span, Associative memory and Associative memory plus conversion are frequently related to achievement of mathematics subtopics at grade four and, with the exception of Memory span, at grade six while these processes contribute to relatively fewer subtopics at grade five. On the other hand, Visualization and Reasoning -- word grouping are more frequently related to subtopics at grades five and six but not at grade four.

Limitations

The conclusions are limited to conditions similar to those involved in the present investigation: i.e., to populations similar to those from which the subjects were selected; to aspects of mathematics achievement and intellectual processes similar to those assessed; to instructional programs at the fourth, fifth, and sixth grades similar to those used to teach mathematics to the populations sampled; and to mathematics achievement sampled at the completion of an instructional program at a given grade level rather than during the instructional program. In addition, the results needed to be further verified

in the study in which the mathematics subtopic scores are based on more items and are characterized by higher reliability than the subtopic scores employed in the present study.

Implications

The basic underlying interest in the present study was primarily heuristic: to obtain information about psychological components of mathematics achievement in order to specify a relational bridge between two domains -- (1) mathematics education and (2) psychological research on learning and intelligence. This endeavor was considered educationally significant in the sense that identifying psychological components of mathematics achievement could be a useful preparatory step in the process of using psychological research as a partial basis for decision making about specific mathematics education procedures.

The results of the study indicated that the predominant psychological components of the facets of mathematics achievement sampled were facility in numerical reasoning and facility in discerning verbal meaning while facility in verbal conceptualization, associative memory, associative memory plus conversion, memory span, and perceptual discrimination were more minor components.

The results of the present study can be viewed with some confidence for at least two reasons: these results are not inconsistent with data reported by previous investigators who employed samples from other populations and who studied diverse aspects of mathematics; these results are understandable in the context of the content and methodology of contemporary mathematics education. Nevertheless, because of the identified limitations to internal and external validity, these results need to be verified and extended. Should these results remain consistent, the next task is to derive implications of pertinent psychological research which can be used as a partial basis for verifying or revising current pedagogical practices and for devising new pedagogical practices. That is, the research literature is replete with information about variables and conditions which affect childrens' reasoning, verbalization, conceptualization, memory, discrimination, and so on. Similarly, more information is being obtained about these

topics currently. A next step is a series of applied research projects and development projects designed to derive and field test the implications of such psychological data and other pertinent considerations for variations in the scope, sequence, timing, methods, and materials of a mathematics instructional program encompassing mathematics topics and subtopics at grades four, five and six like those which were the focus of the present study.

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APPENDICES

APPENDIX A
COOPERATING SCHOOL PERSONNEL

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Administrative Personnel

Superintendent: Mr. Sam W. Wood; Associate Superintendent:
Mr. Alton A. Ellis; Guidance and Testing Director: Mr. J. C. Mullis.

Principals

Miss Allea Betts, Miss Dorothy Firor, Mr. Robert Garrard,
Miss Grace Hancock, Mr. Don Hight, Mrs. Rosa Tarpley, Miss Helen Treanor,
and Miss Annie Wallace.

Teachers

Mrs. Vera Bennett, Mrs. Julia Brackett, Mrs. Marjorie Brightwell,
Mrs. Eva Brown, Mrs. Marlene Bush, Mrs. Margaret Callahan,
Mrs. Eugenia Coggin, Mrs. Beth Cooper, Mrs. Carolyn Davis,
Mrs. Hazel Davis, Mrs. Clara Doster, Mrs. Louise Eidson,
Miss Frances Goodwin, Miss Mildred Johnson, Miss Vada Kent,
Mrs. Mary McCutchen, Mrs. Doris Madden, Mrs. Gertrude Martin,
Mrs. Editha Mills, Mrs. Florrie Oldham, Mrs. Marguerite Patat,
Mrs. Lou Plant, Mrs. Diane Shortall, Mrs. Lily Tabor, and
Miss Annette Zuber.

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APPENDIX B

INSTRUCTIONAL OBJECTIVES AND SELECTED TEST ITEMS

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Explanatory Note

This appendix consists of instructional objectives and test items classified by mathematics topics and subtopics. For each subtopic, the instructional objectives are presented first. In this presentation, the symbol X is used to indicate that an instructional objective is pertinent at a given grade level. When an instructional objective is not pertinent at a given grade level, the abbreviation NA is used to indicate "not applicable". After the instructional objectives are presented, the test items selected to sample achievement of these objectives are listed.

Topic: Number, operation, assumptions

Subtopic: Addition

Instructional Objectives	Grade Level		
	4	5	6
1. To add 2, 3, and 4 place numbers	X	X	X
2. To add columns of 1, 2, 3, and 4 place numbers	X	X	X

$$\begin{array}{r} 29 \\ + 17 \\ \hline \end{array}$$

- F 12
- G 36
- H 43
- J 56

K None of these

$$\begin{array}{r} 28 \\ 59 \\ 94 \\ \hline \end{array}$$

$$+ 96$$

- A 256
- B 257
- C 267
- D 277

E None of these

$$\begin{array}{r} 1111 \\ + 999 \\ \hline \end{array}$$

- A 2000
- B 2010
- C 2100
- D 2110

E None of these

Joan picked 23 tomatoes from her garden on Monday, 36 on Tuesday, and 18 on Wednesday. How many tomatoes did she pick in all during the three days?

- F 67
- G 77
- H 86
- J 87
- K 96

Topic: Number, operation, assumptions

Subtopic: Subtraction

Instructional Objectives	Grade Level		
	4	5	6
1. To subtract 2, 3, and 4 place numbers with regrouping	X	X	X

$$\begin{array}{r} 6007 \\ -3928 \\ \hline \end{array}$$

- A 179
- B 1079
- C 2079
- D 2179
- E None of these

$$\begin{array}{r} 17 \\ - 9 \\ \hline \end{array}$$

- A 6
- B 8
- C 9
- D 12
- E None of these

$$\begin{array}{r} 821 \\ -514 \\ \hline \end{array}$$

- A 207
- B 307
- C 310
- D 317
- E None of these

Bill weighs 109 pounds.
George weighs 127 pounds.
How many pounds difference
is there in their weights?

- F 18
- G 20
- H 28
- J 118
- K 136

Topic: Number, operations, assumptions

Subtopic: Multiplication

Instructional Objectives	Grade Level		
	4	5	6
1. To multiply using 1, 2, and 3 place multipliers, including zeros and regrouping.	X	X	X

800
x 60

- F 480
- G 4,800
- H 48,000
- J 480,000
- K None of these

184
x 25

- F 1,288
- G 3,772
- H 4,600
- J 37,720
- K None of these

580
x 304

- A 4,060
- B 19,720
- C 176,020
- D 176,320
- E None of these

The school bus makes three round trips each day. How many round trips does the bus make in 260 days?

- A 263
- B 520
- C 680
- D 780
- E 980

Topic: Number, operations, assumptions

Subtopic: Division

Instructional Objectives	Grade Level		
	4	5	6
1. To divide by 1 and 2 digit numbers with and without remainders	X	X	X

$$5 \overline{)355}$$

F 61

G 70

H 71

J 710

K None of these

$$3 \overline{)1821}$$

A 60

B 67

C 670

D 1607

E None of these

$$89 \overline{)8366}$$

A 9.4

B 94

C 904

D 940

E None of these

$$32 \overline{)165}$$

F 5

G $5 \frac{5}{32}$

H $5 \frac{15}{32}$

J $6 \frac{5}{32}$

K $6 \frac{25}{32}$

Topic: Number, operations, assumptions

Subtopic: Laws and generalizations

Instructional Objectives	Grade Level		
	4	5	6
1. To demonstrate understanding of generalizations concerning 1 and 0	X	X	X
2. To demonstrate understanding of addition and subtraction as inverse processes	X	X	X
3. To demonstrate understanding of the commutative and associative laws	X	X	X

$$6 \times \square = 0$$

Which one of the numerals goes in the box to make the sentence true?

- A 0
- B 1
- C 6
- D 10
- E $\frac{1}{6}$

Which sentence expresses the property of associativity in addition?

- A $rs = s + r$
- B $r(s+t) = (r+s)t$
- C $r + s = s + r$
- D $(r+s)+t = r+(s+t)$
- E None of these

The inverse operation of addition is

- F Multiplication
- G Subtraction
- H Division
- J Adding the problem again in the reverse order
- K None of these

Which one of the numerals should go in the box to make the sentence true?

$$8 \times 7 = 7 \times \square$$

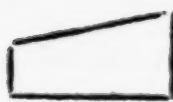
- F 7
- G 8
- H 9
- J 54
- K 56

Topic: Geometry

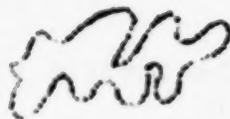
Subtopic: Definitions of Terms

Instructional Objectives	Grade Level		
	4	5	6
1. To define terms referring to parallel lines and angles	X	X	X
2. To define terms referring to circles	NA	X	X
3. To define terms referring to sets of points	NA	X	X

Which figure is not a simple closed curve?



F



G



H



J

- F F J J
 G G K None of these
 H H

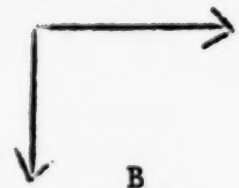
How many endpoints does a line have?

- A Infinitely many
 B Three endpoints
 C Two endpoints
 D One endpoint
 E No endpoints

Which angle is a right angle?



A



B



C



D

- A A D D
 B B E None of these
 C C

The surface of a table may be thought of as

- F a ray
 G a line
 H a square
 J a part of a line
 K a part of a plane

Topic: Geometry

Subtopic: Recognition of geometric figures

Instructional Objectives	Grade Level		
	4	5	6
1. To recognize and name two dimensional (plane) figures	X	X	X
2. To recognize and name three dimensional (solid) figures	NA	X	X

Which figure is a triangle?



F



G



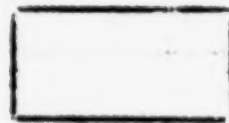
H



J

F F J J
G G K None of these
H H

Which figure is a square?



A



B



C



D

A A D D
B B E None of these
C C

Which figure is a rectangle?



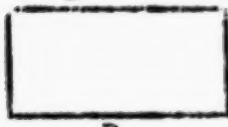
A



B



C



D

A A D D
B B E None of these
C C

Which figure is a cube?



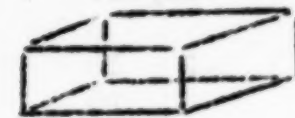
F



G



H



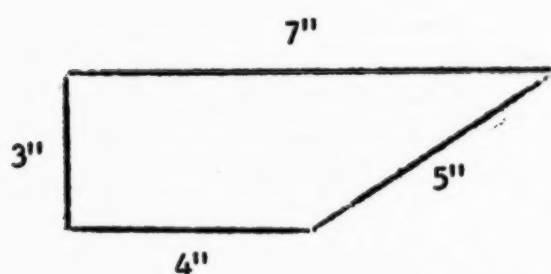
J

F F J J
G G K None of these
H H

Topic: Geometry

Subtopic: Measurement of geometric figures

Instructional Objectives	Grade Level		
	4	5	6
1. To find the perimeter of polygons	NA	X	X
2. To find the area of squares, rectangles, triangles, and similar polygons	NA	X	X
3. To find the volume of rectangular solid	NA	NA	X



What is the measure of the perimeter of the figure above?

- F 15 inches
- G 19 inches
- H 20 square inches
- J 21 square inches
- K 47 inches

For one of her Brownie projects, Janet made a square potholder with each side 10 inches long. About how many inches of tape would she need to go around the edges?

- F 10
- G 20
- H 40
- J 100
- K None of these

A playroom is 30 feet long and 25 feet wide. What is its area in square feet?

- F 55
- G 75
- H 750
- J 1275
- K 1750

What is the volume of a rectangular solid which measures 2 feet in length, 4 feet in width, and 3 feet in height?

- A 8 cu. ft.
- B 12 cu. ft.
- C 20 cu. ft.
- D 24 cu. ft.
- E 36 cu. ft.

Topic: Relations

Subtopic: Equality

Instructional Objectives	Grade Level		
	4	5	6
1. To demonstrate understanding of equivalence	X	X	X
2. To demonstrate understanding of equality	X	X	X

Sue reported to her scout troop one night that there are four planets larger than the earth and four smaller. If the earth has a diameter of 7900 miles, which one of the following planets is smaller than the earth?

- A Venus - diameter 7600 miles
- B Uranus - diameter 30,800 miles
- C Saturn - diameter 72,400 miles
- D Jupiter - diameter 86,700 miles
- E None of these

$$7 + 10 \quad \boxed{} \quad 10 + 7$$

Which symbol should be written in the box to make the sentence true?

- F $<$
- G $>$
- H $\leq$
- J $=$
- K $\sim$

$$10 \quad \boxed{} \quad 9$$

Which symbol should go in the box to make the sentence true?

- A $=$
- B $+$
- C $>$
- D $<$
- E $\leq$

The newspaper said that Chicago had 1.06 inches of rain on Monday, 1.15 inches on Thursday, and 1.51 inches on Saturday. Which day had the LEAST amount of rain?

- F Monday
- G Thursday
- H Saturday
- J Thursday and Saturday
- K You can't tell

Topic: Relations

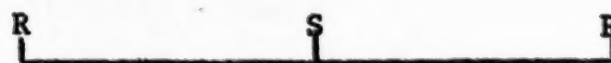
Subtopic: Order relations

Instructional Objectives	Grade Level		
	4	5	6
1. To demonstrate understanding of order relations using a number line	X	X	X
2. To demonstrate understanding of the order of decimals	X	X	X

A boy scout first walked 5 miles east and then 10 miles west. How many miles and in what direction should he walk to get back to his starting point?

- A 5 miles east
- B 5 miles west
- C 15 miles east
- D 15 miles west
- E He is already at his starting point.

Robert's birth is marked on the time line below at R. His brother Sam's birth is marked at S, and the present is marked at P. S is mid-way between R and P.



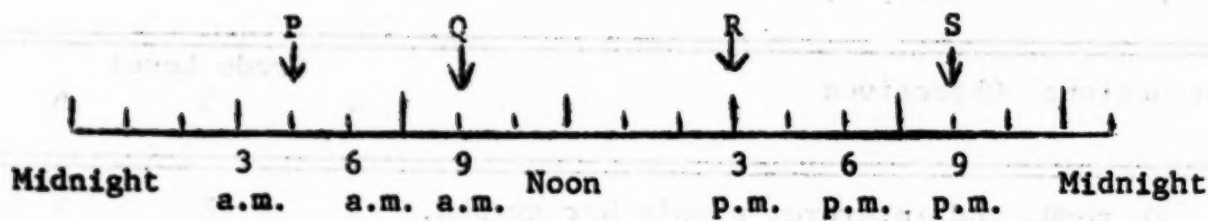
Which one of the following sentences is true?

- F Robert is younger than Sam.
- G Sam and Robert are exactly the same age.
- H Sam is now twice as old as Robert.
- J Robert is now twice as old as Sam.
- K None of these

Which number has the largest value?

- A 2.005
- B 2.05
- C 2.5
- D .25
- E 25

Order Relations (cont.)



An orderly way to think about events is to arrange them along a time line similar to the one above.

Which one of the following points in Mary's day is NOT shown at the most likely place on the time line above?

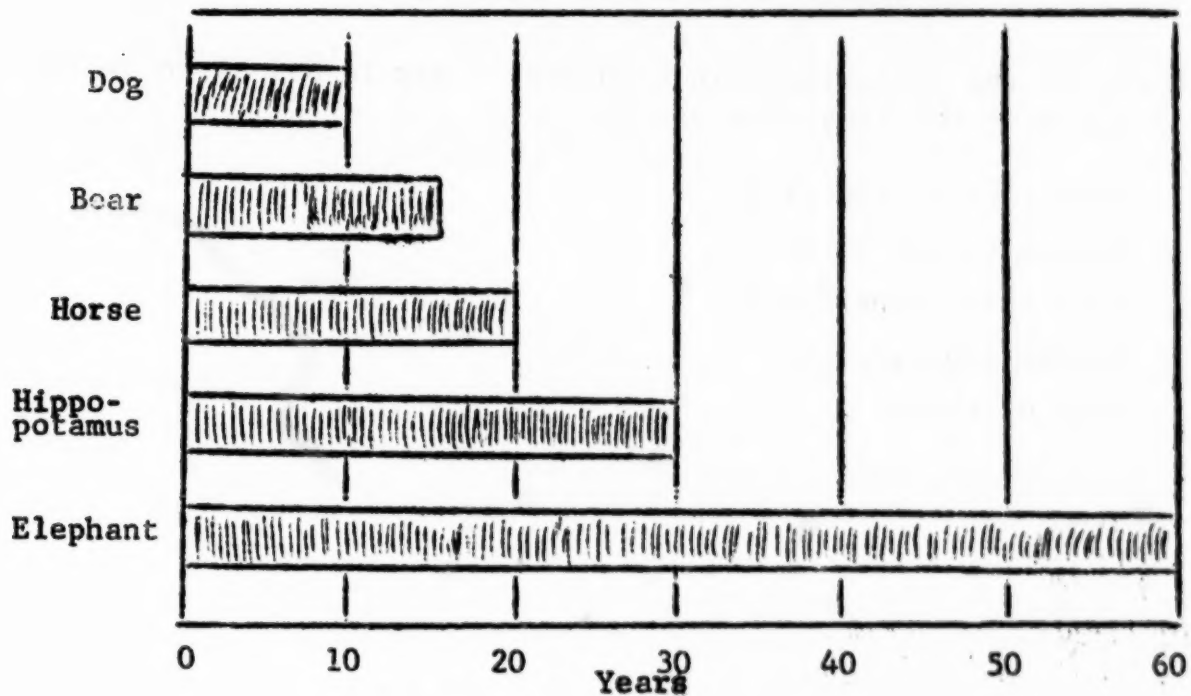
- A Mary goes to bed at S
- B School is out at R
- C Mary eats dinner at P
- D School starts at Q
- E None of these

Topic: Relations

Subtopic: Maps, graphs, charts

Instructional Objectives	Grade Level		
	4	5	6
1. To read, and interpret simple bar graphs.	X	X	X

AVERAGE LIFE SPAN OF ANIMALS



Jim said, "No elephant can live to be 70 years old." Bob said, "All elephants live to be at least 50 years old." Tom said, "All elephants live to be 90 years old." According to the graph, who is right?

F Only Jim

J Both Jim and Bob

G Only Bob

K Neither Jim nor Bob nor Tom

H Only Tom

On the average, how many years does a horse live?

A 2

D 47

B 10

E 57

C 20

On the average, a bear lives how many times as long as a dog?

F 1 1/2

J 10

G 2

K 15

H 5

On the average, which animal lives two times as long as a dog?

A Bear

D Elephant

B Horse

E None of these

C Hippopotamus

Topic: Relations

Subtopic: Functions

Instructional Objectives	Grade Level		
	4	5	6
1. To use ratio in solving problems that involve the idea of rate or comparison	X	X	X
2. To solve problems which involve the concept of proportion	X	X	X

Bill's mother planned to use up a big cake at her son's birthday party by cutting equal pieces for his 20 friends. It rained that day and only 10 children were at the party. Bill's mother still planned to use up the whole cake by cutting equal pieces for the 10 children instead. This meant that each serving was

- A two times as large as she had planned
- B one-fourth as large as she had planned
- C one-half as large as she had planned
- D four times as large as she had planned
- E unequal in size

On a certain map 2 inches stands for one mile. How many inches on the map would stand for a distance of 10 miles?

- A 5 C 10 E 20
- B 8 D 15

John saves 10 cents a week. Susan saves 15 cents a week. At this rate after how many weeks will Susan have saved 5 times as much as John?

- F 1 H 3 K Never
- G 2 J 5

Bill and Sarah had a bicycle race from one town to another. Bill took one hour and twenty minutes. Sarah took 80 minutes. Bill's speed was

- F $\frac{4}{3}$ of Sarah's speed
- G $\frac{3}{2}$ of Sarah's speed
- H $\frac{3}{4}$ of Sarah's speed
- J $\frac{2}{3}$ of Sarah's speed
- K the same as Sarah's speed

Topic: Numeration

Subtopic: Sets

Instructional Objectives	Grade Level		
	4	5	6
1. To demonstrate an understanding of the concept of sets and set terminology	X	X	X
2. To distinguish among kinds of sets	X	X	X
3. To demonstrate an understanding of union and intersection of sets	X	X	X

A symbol for an empty set is

F $<$

G $\{$

H $\simeq$

J $>$

K $\emptyset$

The union of $\{2, 4, 6\}$ and $\{3, 6, 9\}$ is

A $\{2, 4, 6, 6\}$

B $\{2, 3, 4, 6, 6, 9\}$

C $\{6\}$

D $\{2, 3, 4, 6, 9\}$

E None of these

A set which is equal to $\{3, 5, 11, 7, 9\}$ is

F $\{35\}$

G $\{2, 4, 10, 6, 8\}$

H $\{11, 9, 7, 5, 3\}$

J $\{3, 5, 7, 9\}$

K None of these

If Set C = $\{6, 7, 8, 9\}$ and Set D = $\{4, 5, 6, 7\}$ find Set $C \cap D$.

A $\{6, 7, 8\}$

B $\{7, 8, 9\}$

C $\{6, 7\}$

D $\{6, 9, 4, 7\}$

E $\{4, 5, 7, 8, 9\}$

Topic: Numeration

Subtopic: Other Numeration Bases

Instructional Objectives	Grade Level		
	4	5	6
1. To compare base five and base ten	X	X	X
2. To count beyond the square of the base	X	X	X
3. To use other number bases	X	X	X

23_{five} means

- A twenty-three
- B 2 fives and 3 ones
- C 2 tens and 3 ones
- D 2 tens and 3 fives
- E 23 fives and 0 ones

13 14 15 20 21 22

In what number base is the counting above done?

- F Base_{three}
- G Base_{five}
- H Base_{six}
- J Base_{ten}
- K Base_{sixteen}

Which is the largest two-place numeral that can be written in base five?

- A 44_{five}
- B 55_{five}
- C 66_{five}
- D 99_{five}
- E None of these

What numeral in base ten stands for the same number as 440_{five} ?

- F 44_{ten}
- G 84_{ten}
- H 88_{ten}
- J 120_{ten}
- K 220_{ten}

Topic: Numeration

Subtopic: Reading, writing, using whole numbers and decimals

Instructional Objectives	Grade Level		
	4	5	6
1. To read, write, and use whole numbers to nine places	X	X	X
2. To read, write, and use decimals	X	X	X

In the number 5.05, the digit 5 is used twice. Which one of the following is true?

- A The "5's" stand for the same amount.
- B The "5" at the left stands for an amount 10 times the amount represented by the "5" on the right.
- C The "5" at the right stands for an amount larger than the one on the left.
- D The "5" at the left stands for an amount 100 times the amount represented by the "5" on the right.
- E The "5" at the left stands for an amount 1000 times the amount represented by the "5" on the right.

The numeral 643 has three digits. What does the "6" stand for?

- A Six
- B Six tens
- C The number of hundreds
- D The number of six-hundreds
- E The number of thousands

The school district paid sixty thousand dollars for new buses. How would you write this amount?

- F \$60
- G \$600
- H \$6,000
- J \$60,000
- K \$600,000

Which numeral stands for the smallest number you can write with the digits 4, 9, 1, and 6?

- F 1946
- G 9164
- H 1469
- J 1649
- K 4961

Topic: Numeration**Subtopic: Roman Numerals**

Instructional Objectives	Grade Level		
	4	5	6
1. To read and write Roman Numerals through LXXX	X	X	X

In the Cathedral Latin School, one of the rooms is numbered XVI. What does XVI mean?

- A 10 hundreds + 5 tens + 1
- B 10 + 5 + 1
- C 10 + 4
- D 151
- E 61

In Roman Numerals, what letter do we place before X to make 9?

- A L
- B D
- C I
- D C
- E X

XLV is the same as

- F 25
- G 45
- H 64
- J 615
- K 1054

In Roman numerals, what letter do we place after X to make 19?

- F XI
- G IX
- H LXX
- J LX
- K XXI

Topic: Measurement

Subtopic: Concepts

Instructional Objectives	Grade Level		
	4	5	6
1. To demonstrate an understanding of measurement concepts related to the following: size, quantity, distance, time, and weight.	X	X	X

Which unit is a measure of length?

- A A square foot
- B An acre
- C A square yard
- D A mile
- E A cubic yard

Which unit of measurement is found in the metric system?

- F A pint
- G A yard
- H A meter
- J A ton
- K A week

A half-pint is related to a pint the way a

- F pint is related to a quart
- G pint is related to a gallon
- H quart is related to a gallon
- J pint is related to a half gallon
- K quart is related to five gallons

John is in school every weekday from 8:30 a.m. until 2:45 p.m. How many hours does he spend in school each day?

- F 5 1/4
- G 5 3/4
- H 6
- J 6 1/4
- K 7 1/4

Topic: Measurement

Subtopic: Conversion of units

Instructional Objectives	Grade Level		
	4	5	6
1. To change measures from one unit to another	X	X	X

John helped clear fallen trees after a storm. He saw a large tree which he guessed was 20 yards long. The tree was actually 65 feet long. John's guess

F was too small

G was exactly right

H was too large

J cannot be compared to the actual length

K was none of these

Betty practices her music lessons 140 minutes each week. How much time will she spend practicing in 4 weeks?

A 3 hours 20 minutes

B 4 hours 20 minutes

C 8 hours 20 minutes

D 9 hours 20 minutes

E 9 hours 80 minutes

Betty has been saving pennies in a jar. She found that she had six hundred thirty-five pennies. This is the same as

A \$6.35

B \$35.60

C \$63.50

D \$600.35

E \$635.00

David is 42 inches tall. This is the same as

F 3 feet

G 3 feet 6 inches

H 3 feet 9 inches

J 4 feet

K 4 feet 2 inches

Topic: Measurement

Subtopic: Operations without Conversion

Instructional Objectives	Grade Level		
	4	5	6
1. To solve problems using operations with measures	X	X	X

On a week-end trip George and his father drove 97.3 miles on Friday, 104.8 miles on Saturday, and 124.6 miles on Sunday. How many miles did they drive on the whole trip?

A 215.7

D 2157

B 226.7

E 3667

C 326.7

Amy is 56 inches tall. Joan is 50 inches tall. Amy is how many inches taller than Joan?

If erasers cost 5 cents each, how many cents do 7 erasers cost?

A 6

F 5

B 16

G 7

C 50

H 12

D 56

J 35

E 106

K 45

Janet's mother bought $\frac{1}{2}$ of a quart of peach ice cream and $1\frac{1}{2}$ quarts of chocolate ice cream for Janet's party. How many quarts of ice cream did she buy in all?

A $\frac{3}{4}$

B 1

C 2

D 3

E 4

Topic: Measurement

Subtopic: Operations with Conversion

Instructional Objectives	Grade Level		
	4	5	6
1. To solve problems which require conversion of units of measure, using all operations	X	X	X

Bill bought 3 pounds of bananas at 8 cents per pound. Which one of the following groups of coins would be the right amount to pay for the bananas?

- | | |
|--------------------------------|---------------------------------|
| F 1 dime, 2 nickels, 4 pennies | J 2 dimes, 2 nickels, 4 pennies |
| G 1 quarter, 4 pennies | K 3 dimes, 4 pennies |
| H 2 dimes, 1 nickel, 4 pennies | |

The Riley family uses 4 eggs each day. How many days will 3 dozen eggs last the Riley family?

- A 4
- B 9
- C 12
- D 36
- E 40

Ellen was 2'8" tall when David was 3'4". At that time, David was how many inches taller than Ellen?

- F 4
- G 6
- H 8
- J 14
- K 16

Mr. Clark drove his car 1212 miles last year. How many miles on the average did he drive his car each month?

- F 11
- G 101
- H 121.2
- J 1200
- K 1212

Topic: Fundamental operations with fractions

Subtopic: Concept of fractions

Instructional Objectives	Grade Level		
	4	5	6
1. To find common fractional parts of the whole object and the group	X	X	X
2. To demonstrate understanding of a reciprocal	NA	NA	X
3. To interpret decimals as an extension of one number system	NA	X	X

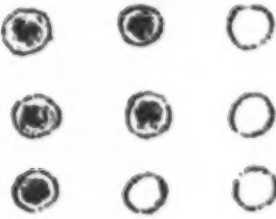
The reciprocal of $\frac{7}{8}$ is

- A 1
B 0
C $\frac{8}{7}$
D $\frac{7}{8}$
E $\frac{49}{64}$

Which decimal numeral names 951 hundredths?

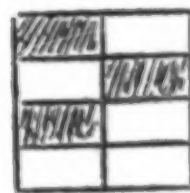
- A .0951
B .951
C 9.51
D 95.1
E 951.

What fraction of the total number of circles is shaded?

- F 5
G $\frac{4}{5}$
H $\frac{4}{9}$
J $\frac{5}{9}$
K 4
- 

What part of the rectangle is shaded?

- F $\frac{1}{4}$
G $\frac{1}{3}$
H $\frac{3}{8}$
J $\frac{3}{5}$
K $\frac{5}{8}$



Topic: Fundamental operations with fractions

Subtopic: Addition

Instructional Objectives	Grade Level		
	4	5	6
1. To add like fractions	X	X	X
2. To add decimals	X	X	X
3. To add unlike but related fractions	NA	X	X

Betty and Carol are making dresses for their dolls. Betty bought $\frac{3}{4}$ of a yard of material and Carol bought $\frac{1}{2}$ of a yard of material. How many yards of material did the girls buy altogether?

F $\frac{2}{3}$

G $\frac{5}{6}$

H 1

J $1\frac{1}{4}$

K $1\frac{2}{3}$

$$\begin{array}{r} \frac{1}{3} \\ + \frac{1}{3} \\ \hline \end{array}$$

F $\frac{1}{2}$

G $\frac{2}{3}$

H $\frac{8}{9}$

J $1\frac{1}{3}$

K None of these

$$\begin{array}{r} 8.09 \\ + 24.76 \\ \hline \end{array}$$

F. 22.75

G. 22.85

H. 32.75

J. 32.85

K. None of these

$$\frac{1}{6} + \frac{1}{6} = (?)$$

F $\frac{1}{3}$

G $\frac{1}{12}$

H $\frac{2}{12}$

J $\frac{1}{36}$

K None of these

Topic: Fundamental operations with fractions

Subtopic: Subtraction

Instructional Objectives	Grade Level		
	4	5	6
1. To subtract like fractions	X	X	X
2. To subtract decimals	X	X	X
3. To subtract unlike but related fractions	NA	X	X

$$\begin{array}{r} 60.00 \\ - 28.74 \\ \hline \end{array}$$

- A 21.26
- B 31.26
- C 42.36
- D 48.74
- E None of these

$$\begin{array}{r} \frac{5}{7} \\ - \frac{2}{7} \\ \hline \end{array}$$

- F $\frac{3}{14}$
- G $\frac{2}{7}$
- H $\frac{3}{7}$
- J 3
- K None of these

$$5 - \frac{2}{5} = (?)$$

A $\frac{3}{5}$

B 2

C $4\frac{3}{5}$

D $5\frac{3}{5}$

E None of these

Philip had $\frac{3}{4}$ of a gallon of gas. If he used $\frac{1}{2}$ of a gallon in his model plane, how many gallons of gas did he have left?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{3}{8}$

D $\frac{1}{2}$

E $1\frac{1}{4}$

Topic: Fundamental operations with fractions

Subtopic: Multiplication

Instructional Objectives	Grade Level		
	4	5	6

1. To multiply a common fraction by a whole number	NA	NA	X
2. To multiply a fraction by a fraction	NA	NA	X
3. To multiply a decimal fraction by a whole number	NA	NA	X
4. To multiply a decimal fraction by a decimal fraction	NA	NA	X

Susan knows the cost of $\frac{1}{3}$ yard of ribbon. To find the cost per yard, she should

A add $\frac{1}{3}$ three timesD divide the cost per yard by $\frac{1}{3}$ B subtract $\frac{1}{3}$ from the cost per yardE divide the cost of $\frac{1}{3}$ yard by threeC multiply the cost of $\frac{1}{3}$ yard by three
$$\begin{array}{r} 1.3 \\ \times .04 \\ \hline \end{array}$$

$$\begin{array}{r} .33 \\ \times 2 \\ \hline \end{array}$$

F .0052

J 5.20

A .066

D 66

G .052

K 52.

B .66

E None of these

H .52

C 6.6

$$\frac{3}{7} \times \frac{4}{5} = (?)$$

F $\frac{12}{35}$ J $\frac{1}{5}$ G $\frac{15}{28}$

K None of these

H $\frac{7}{12}$

Topic: Fundamental operations with fractions

Subtopic: Division

Instructional Objectives	Grade Level		
	4	5	6
1. To divide a whole number by a decimal fraction	NA	NA	X
2. To divide a fraction by a fraction	NA	NA	X

Four pounds and 4 ounces is the same as

- A 4.15 pounds
- B $4\frac{1}{4}$ pounds
- C $4\frac{1}{2}$ pounds
- D $4\frac{3}{4}$ pounds
- E 5 Pounds

Find the correct answer.

$$\frac{2}{5} \div \frac{7}{10} =$$

- A $\frac{4}{7}$
- B $\frac{2}{7}$
- C $\frac{2}{25}$
- D $\frac{7}{25}$
- E $\frac{7}{50}$

Find the correct answer.

$$.4 / \overline{122}$$

- A .035
- B .305
- C 3.05
- D 30.5
- E 305

Which sentence is another way to express the relationship

$$\frac{\frac{3}{4}}{\frac{5}{8}} = n?$$

- F $\frac{3}{4} \div \frac{8}{5} = n$
- G $\frac{3}{4} \div \frac{5}{8} = n$
- H $\frac{4}{3} \times \frac{5}{8} = n$
- J $\frac{4}{3} \times \frac{8}{5} = n$
- K $\frac{3}{4} - \frac{5}{8} = n$

APPENDIX C

SUPPLEMENTARY MATHEMATICS TEST

SUPPLEMENTARY MATHEMATICS TEST

DIRECTIONS FOR EXAMINER

(Repeat the directions in the SCAT examiners's manual on pages 6 and 7 as you did for SCAT Parts II and IV. When you finish the General Directions Section, say:)

On the front of your booklet, you will find Directions for Part I of the test. Read the directions silently while I read them aloud.

There are 30 problems in Part I of the test. Following each problem there are five suggested answers. Work each problem in your head or on a piece of scratch paper. Then look at the five suggested answers and decide which one is correct. Blacken the space under its letter on the answer sheet.

Sample Problem

4251 A 5527 B 5627 C 2875 D 5628 E None of these
+ 1376

Because the correct answer to the sample problem is 5627, which is lettered B, the space marked B on the answer sheet is blackened. See how it is marked on the answer sheet. Do not make any marks in your test booklet. Do not turn the page until you are told to do so. (Be sure pupils are marking their answers in the correct sections of the answer sheet.)

Are there any questions about what you should do in Part I?

(Pause momentarily to allow questions. Then say:)

There are 30 problems in Part I and you will have 25 minutes to work them. When I say "Begin", turn to the next page and start to work. Ready? Begin.

(Move quietly around the room to make sure that every student is working in the proper part of the test and the answer sheet. At the end of 25 minutes, say:)

Stop! Even if you have not finished Part I you must stop and lay down your pencil.

Turn the page to the directions for Part III.

Read the directions silently while I read them aloud.

There are 30 problems in Part III of the test. Following each problem there are five suggested answers. Work each problem in your head or on a piece of scratch paper. Then look at the five suggested answers and decide which one is correct. Blacken the space under its letter on the answer sheet.

Sample Problem

Jane spent 25 ¢ for candy and 49¢ for a fountain pen. How much did she spend in all?

- A 64¢ B 75¢ C 73¢
D 74¢ E None of these .

Because the correct answer to the sample problem is 74¢, which is lettered D, the space marked D on the answer sheet is blackened. See how it is marked on the answer sheet. Do not make any marks in your test booklet. Do not turn the page until you are told to do so. (Be sure pupils are marking their answers in the correct section of the answer sheet.)

Are there any questions about what you should do in Part III?

(Pause momentarily to allow questions. Then say:)

There are 30 problems in Part III and you will have 30 minutes to work them. When I say "Begin", turn to the next page and start to work. Ready? Begin!

(Move quietly around the room to make sure that every student is working in the proper part of the test and answer sheet. At the end of 30 minutes, say:)

STOP! Even if you have not finished Part III you must stop and lay down your pencil.

(Collect test books, answer sheets, and IBM pencils.)

MATHEMATICS-PART I

DIRECTIONS

There are 30 problems in Part I of the test. Following each problem there are five suggested answers. Work each problem in your head or on a piece of scratch paper. Then look at the five suggested answers and decide which one is correct. Blacken the space under its letter on the answer sheet.

Sample Problem

4251 A 5527 B 5627 C 2875 D 5628 E None of these
+ 1376

Because the correct answer to the sample problem is 5627, which is lettered B, the space marked B on the answer sheet is blackened. See how it has been marked on the answer sheet. Do not make any marks in your test booklet.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

PART I / TIME: 25 MINUTES

- 1 Betty has been saving pennies in a jar. She found that she had six hundred thirty-five pennies. This is the same as

A \$6.35
B \$35.60
C \$63.50
D \$600.35
E \$635.00

- 2 Which numeral stands for the smallest number you can write with the digits 4, 9, 1, and 6?

F 1946
G 9164
H 1469
J 1649
K 4961

- 3 Which number has the largest value?

A 2.005
B 2.05
C 2.5
D .25
E 25

- 4 The school district paid sixty thousand dollars for new buses. How would you write this amount?

F \$60
G \$600
H \$6,000
J \$60,000
K \$600,000

- 5 Which decimal numeral names 951 hundredths?

A .0951
B .951
C 9.51
D 95.1
E 951.

$$7 + 10 \square 10 + 7$$

Which symbol should be written in the box to make the sentence true?

F <
G >
H ≤
J =
K ≈

$$10 \square 9$$

Which symbol should go in the box to make the sentence true?

A =
B +
C >
D <
E ≤

8

$$8 \times 7 = 7 \times \square$$

Which one of the numerals should go in the box to make the sentence true?

F 7
G 8
H 9
J 54
K 56

9

$$6 \times \square = 0$$

Which one of the numerals goes in the box to make the sentence true?

A 0
B 1
C 6
D 10
E $\frac{1}{6}$

GO ON TO THE NEXT PAGE

- 10 In Roman numerals, what letters are placed after X to make 19?
- F XI
G IX
H IXX
J LX
K XXI
- 11 In Roman numerals, what letter is placed before X to make 9?
- A L
B D
C I
D C
E X
- 12 XLV is the same as
- F 25
G 45
H 64
J 615
K 1054
- 13 In the Cathedral Latin School, one of the rooms is numbered XVI. What does XVI mean?
- A 10 hundreds + 5 tens + 1
B 10 + 5 + 1
C 10 + 4
D 151
E 61
- 14 Find the sum:
- $$\begin{array}{r} 6 \\ 1 \\ 9 \\ \underline{5} \end{array}$$
- F 11
G 12
H 21
J 22
K 31
- 15 The numeral 643 has three digits. What does the "6" stand for?
- A Six
B Six tens
C The number of hundreds
D The number of six-hundreds
E The number of thousands
- 16 The inverse operation of addition is
- F Multiplication
G Subtraction
H Division
J Adding the problem again in reverse order
K None of these
- 17 In the number 5.05, the digit 5 is used twice. Which one of the following is true?
- A The "5's" stand for the same amount.
B The "5" at the left stands for an amount 10 times the amount represented by the "5" on the right.
C The "5" at the right stands for an amount larger than the one on the left.
D The "5" at the left stands for an amount 100 times the amount represented by the "5" on the right.
E The "5" at the left stands for an amount 1000 times the amount represented by the "5" on the right.

GO ON TO THE NEXT PAGE.

- 18 The newspaper said that Chicago had 1.06 inches of rain on Monday, 1.15 inches on Thursday, and 1.51 inches on Saturday. Which day had the LEAST amount of rain?

F Monday
G Thursday
H Saturday
J Thursday and Saturday
K You can't tell

- 19 The correct answer to the division problem below is

$$7 \overline{) 630}$$

A 7
B 9
C 19
D 70
E None of these

- 20 Which unit of measurement is found in the metric system?

F A pint
G A yard
H A meter
J A ton
K A week

- 21 Which sentence expresses the property of associativity in addition?

A $rs = s+r$
B $r(s+t) = (r+s)t$
C $r + s = s + r$
D $(r+s)+t = r+(s+t)$
E None of these

- 22 A symbol for an empty set is

F $<$
G $\{$
H $\approx$
J $>$
K $\emptyset$

- 23 The union of $\{2,4,6\}$ and $\{3,6,9\}$ is

A $\{2,4,6,6\}$
B $\{2,3,4,6,6,9\}$
C $\{6\}$
D $\{2,3,4,6,9\}$
E None of these

- 24 A set which is equal to $\{3,5,11,7,9\}$ is

F $\{35\}$
G $\{2,4,10,6,8\}$
H $\{11,9,7,5,3\}$
J $\{3,5,7,9\}$
K None of these

- 25 If set C = $\{6,7,8,9\}$ and Set D = $\{4,5,6,7\}$ find Set C $\cap$ D.

A $\{6,7,8\}$
B $\{7,8,9\}$
C $\{6,7\}$
D $\{6,9,4,7\}$
E $\{4,5,6,7,8,9\}$

- 26 David is 42 inches tall. This is the same as

F 3 feet
G 3 feet 6 inches
H 3 feet 9 inches
J 4 feet
K 4 feet 2 inches

- 27 Which unit is a measure of length?

A A square foot
B An acre
C A square yard
D A mile
E A cubic yard

GO ON TO THE NEXT PAGE.

- 28 Robert's birth is marked on the time line below at R. His brother Sam's birth is marked at S, and the present is marked at P. S is midway between R and P.



Which one of the following sentences is true?

- F Robert is younger than Sam.
- G Sam and Robert are exactly the same age.
- H Sam is now twice as old as Robert.
- J Robert is now twice as old as Sam.
- K None of these

- 30 What fraction of the total number of circles is shaded?



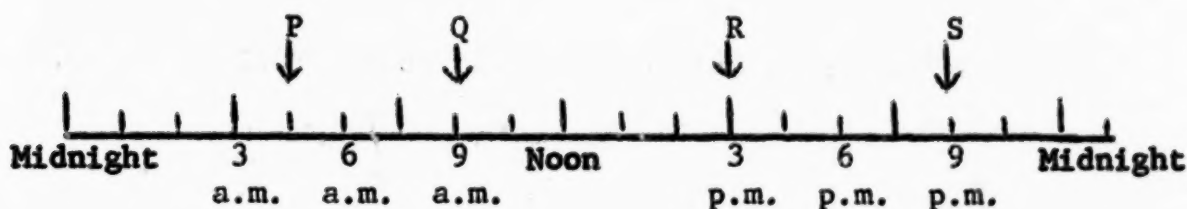
F 5

G $\frac{4}{5}$

H $\frac{4}{9}$

J $\frac{5}{9}$

K 4



- 29 An orderly way to think about events is to arrange them along a time line similar to the one above.

Which one of the following points in Mary's day is NOT shown at the most likely place on the time line above?

- A Mary goes to bed at S.
- B School is out at R.
- C Mary eats dinner at P.
- D School starts at Q.
- E None of these

STOP. If you finish before time is called, check your work on this part. Do not go on to the next part until you are told to do so.

MATHEMATICS-PART III

DIRECTIONS

There are 30 problems in Part III of the test. Following each problem there are five suggested answers. Work each problem in your head or on a piece of scratch paper. Then look at the five suggested answers and decide which one is correct. Blacken the space under its letter on the answer sheet.

Sample Problem

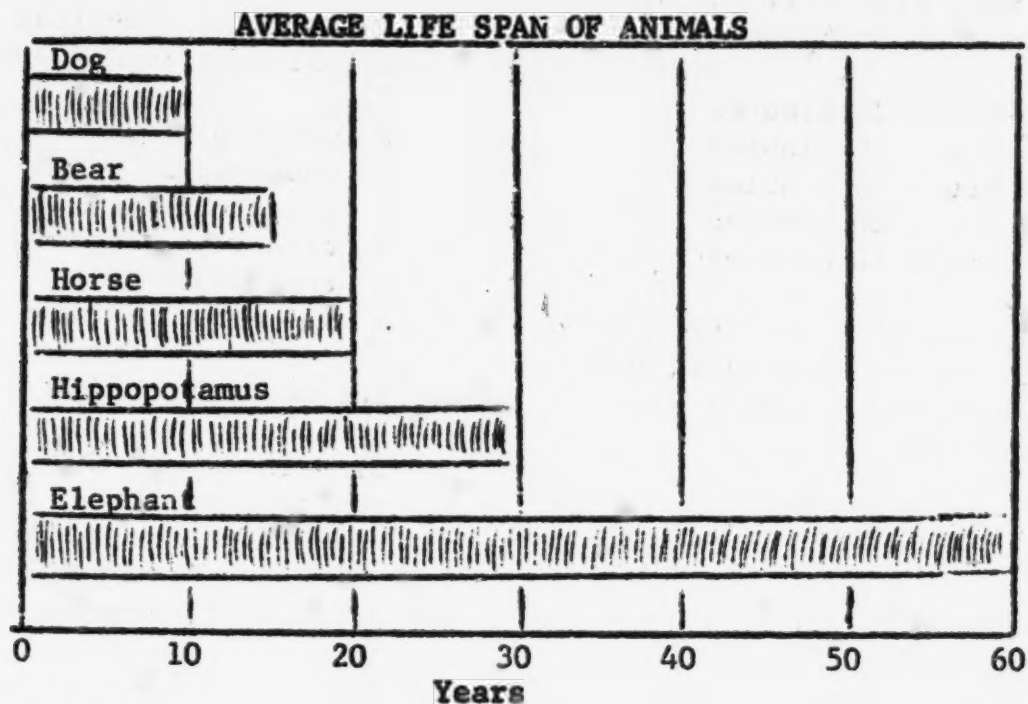
Jane spent 25¢ for candy and 49¢ for a fountain pen. How much did she spend in all?

A 64¢ B 75¢ C 73¢
D 74¢ E None of these

Because the correct answer to the sample problem is 74¢, which is lettered D, the space marked D on the answer sheet is blackened. See how it has been marked on the answer sheet. Do not make any marks in your test booklet.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

PART III /TIME: 30 MINUTES



- 1 On the average, how many years does a horse live?
 - A 2
 - B 10
 - C 20
 - D 47
 - E 57
- 2 On the average, a bear lives how many times as long as a dog?
 - F $1\frac{1}{2}$
 - G 2
 - H 5
 - J 10
 - K 15
- 3 On the average, which animal lives two times as long as a dog?
 - A Bear
 - B Horse
 - C Hippopotamus
 - D Elephant
 - E None of these
- 4 Jim said, "No elephant can live to be 70 years old." Bob said, "All elephants live to be at least 50 years old." Tom said, "All elephants live to be 90 years old." According to the graph, who is right?
 - F Only Jim
 - G Only Bob
 - H Only Tom
 - J Both Jim and Bob
 - K Neither Jim nor Bob nor Tom

GO ON TO THE NEXT PAGE

- 5 Betty practices her music lessons 140 minutes each week. How much time will she spend practicing in 4 weeks?
- A 3 hours 20 minutes
B 4 hours 20 minutes
C 8 hours 20 minutes
D 9 hours 20 minutes
E 9 hours 80 minutes
- 6 Ellen was 2'8" tall when David was 3'4". At that time, David was how many inches taller than Ellen?
- F 4
G 6
H 8
J 14
K 16
- 7 Four pounds and 4 ounces is the same as
- A 4.15 pounds
B $4\frac{1}{4}$ pounds
C $4\frac{1}{2}$ pounds
D $4\frac{3}{4}$ pounds
E 5 pounds
- 8 The surface of a table may be thought of as
- F a ray
G a line
H a square
J a part of a line
K a part of a plane
- 9 How many endpoints does a line have?
- A Infinitely many
B Three endpoints
C Two endpoints
D One endpoint
E No endpoints
- 10 John helped clear fallen trees after a storm. He saw a large tree which he guessed was 20 yards long. The tree actually was 65 feet long. John's guess
- F was too small
G was exactly right
H was too large
J cannot be compared to the actual length
K was none of these
- 11 Sue reported to her scout troop one night that there are four planets larger than the earth and four smaller. If the earth has a diameter of 7900 miles, which one of the following planets is smaller than the earth?
- A Venus - 7600 miles
B Uranus - 30,800 miles
C Saturn - 72,400 miles
D Jupiter - 86,700 miles
E None of these
- 12 For one of her Brownie projects, Janet made a square potholder with each side 10 inches long. About how many inches of tape would she need to go around the edges?
- F 10
G 20
H 40
J 100
K None of these
- 13 Which is the largest two-place numeral that can be written in base five?
- A 44_{five}
B 55_{five}
C 66_{five}
D 99_{five}
E None of these

GO ON TO NEXT PAGE.

- 14 What numeral in base ten stands for the same number as

440_{five}?

- F 44_{ten}
G 84_{ten}
H 88_{ten}
J 120_{ten}
K 220_{ten}

- 15 23_{five} means which of the following?

- A twenty-three
B 2 fives and 3 ones
C 2 tens and 3 ones
D 2 tens and 3 fives
E 23 fives and 0 ones

- 16 A playroom is 30 feet long and 25 feet wide. What is its area in square feet?

- F 55
G 75
H 750
J 1275
K 1750

- 17 Find the correct answer.

$$.4 / 122$$

- A .035
B .305
C 3.05
D 30.5
E 305.

- 18 Find the correct answer.

$$\begin{array}{r} 1.3 \\ \times .04 \\ \hline \end{array}$$

- F .0052
G .052
H .52
J 5.20
K 52.

- 19 Find the correct answer.

$$\frac{2}{5} \div \frac{7}{10} =$$

- A $\frac{4}{7}$
B $\frac{2}{7}$
C $\frac{2}{25}$
D $\frac{7}{25}$
E $\frac{7}{50}$

- 20 Which sentence is another way to express the relationship

$$\frac{\frac{3}{4}}{\frac{5}{8}} = n?$$

- F $\frac{3}{4} \div \frac{8}{5} = n$
G $\frac{3}{4} \div \frac{5}{8} = n$
H $\frac{4}{3} \times \frac{5}{8} = n$
J $\frac{4}{3} \times \frac{8}{5} = n$
K $\frac{3}{4} - \frac{5}{8} = n$

- 21 The reciprocal of $\frac{7}{8}$ is

- A 1
B 0
C $\frac{8}{7}$
D $\frac{7}{8}$
E $\frac{49}{64}$

GO ON TO NEXT PAGE.

22 13 14 15 20 21 22

In what number base is the counting above done?

F Base three

G Base five

H Base six

J Base ten

K Base sixteen

23 What is the volume of a rectangular solid which measures 2 feet in length 4 feet in width, and 3 feet in height?

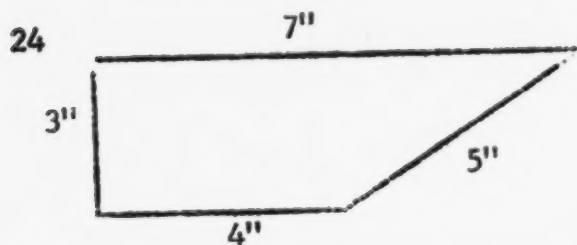
A 8 cubic feet

B 12 cubic feet

C 20 cubic feet

D 24 cubic feet

E 36 cubic feet



What is the measure of the perimeter of the figure above?

F 15 inches

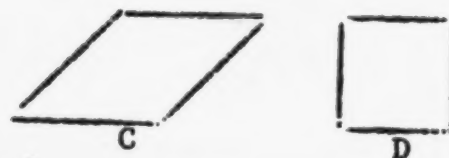
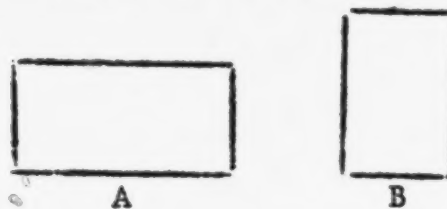
G 19 inches

H 20 square inches

J 21 square inches

K 47 inches

25 Which figure is a square?



A A

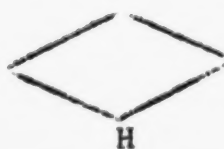
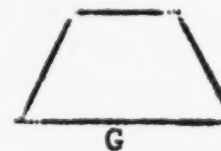
B B

C C

D D

E None of these

26 Which figure is a triangle?



F F

G G

H H

J J

K None of these

GO ON TO THE NEXT PAGE.

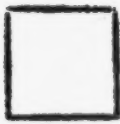
27 Which figure is a rectangle?



A



C



B



D

A A D D
 B B E None of these
 C C

28 Which figure is a cube?



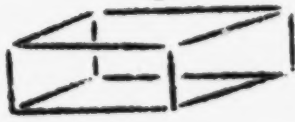
F



H



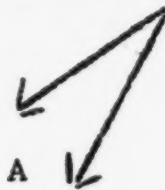
G



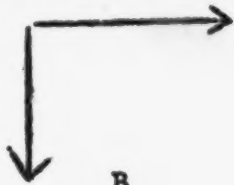
J

F F
 G G
 H H
 J J
 K None of these

29 Which angle is a right angle?



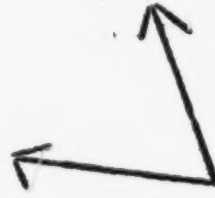
A



B



C



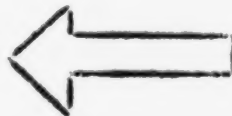
D

A A D D
 B B E None of these
 C C

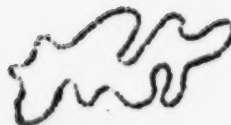
30 Which figure is NOT a simple closed curve?



F



H



G



J

F F
 G G
 H H
 J J
 K None of these

STOP. If you finish before time
 is called check your work
 on this part.

PRECEDING PAGE BLANK-NOT FILMED

APPENDIX D

SOURCE AND IDENTIFICATION OF MATHEMATICS

TEST ITEMS

Introduction

Listed below are the source and identification of the items included in the test. They are organized by topic, subtopic, item identification, and location in the supplementary test. The names of the tests have been abbreviated. The abbreviations used are the following.

SCAT - School and College Ability Tests, Form A

STEP - A - Sequential Tests of Educational Progress, Level 4, Form A

STEP - B - Sequential Tests of Educational Progress, Level 4, Form B

CL - Contemporary Mathematics Tests, Lower Elementary Level, Form A

CU - Contemporary Mathematics Tests, Upper Elementary Level, Form A

Topic INumber, Operations, Assumptions

<u>Subtopics</u>	<u>Item Identification</u>	<u>Part and Item Number: Supplementary Test</u>
A. Addition	SCAT, Pt. II, Item 2 SCAT, Pt. II, Item 3 SCAT, Pt. IV, Item 4 SCAT, Pt. II, Item 5	
B. Subtraction	SCAT, Pt. II, Item 1 SCAT, Pt. II, Item 7 SCAT, Pt. II, Item 11 SCAT, Pt. IV, Item 6	
C. Multiplication	SCAT, Pt. II, Item 14 SCAT, Pt. II, Item 18 SCAT, Pt. II, Item 21 SCAT, Pt. IV, Item 5	
D. Division	SCAT, Pt. II, Item 4 SCAT, Pt. II, Item 19 SCAT, Pt. II, Item 20 SCAT, Pt. II, Item 23	
-		
E. Laws and Generalizations	CL, Item 48 CL, Item 50 CL, Item 48 CL, Item 53	Part I, Item 9 Part I, Item 8 Part I, Item 21 Part I, Item 16

Topic II
Geometry

<u>Subtopics</u>	<u>Item Identification</u>	<u>Part and Item Number: Supplementary Test</u>
A. Definitions of Terms	CL, Item 3 CL, Item 4 CL, Item 26 Author	Part III, Item 9 Part III, Item 8 Part III, Item 30 Part III, Item 29
B. Recognition of Geometric Figures	STEP-A, Pt. I, Item 17 STEP-A, Pt. I, Item 18 STEP-B, Pt. I, Item 9 Author	Part III, Item 26 Part III, Item 25 Part III, Item 27 Part III, Item 28
C. Measurement of Geometric Figures	STEP-A, Pt. I, Item 1 CU, Item 9 Author Author	Part III, Item 12 Part III, Item 24 Part III, Item 23 Part III, Item 16

Topic III
Relations

A. Equality	STEP-A, Pt. II, Item 12 STEP-B, Pt. II, Item 3 CU, Item 14 CU, Item 42	Part III, Item 11 Part I, Item 18 Part I, Item 7 Part I, Item 6
B. Order Relationships	SCAT, Pt. IV, Item 25 STEP-B, Pt. I, Item 21 STEP-B, Pt. I, Item 25 Author	Part I, Item 29 Part I, Item 28 Part I, Item 3
C. Map, Graphs, Charts	STEP-B, Pt. II, Item 10 STEP-B, Pt. II, Item 11 STEP-B, Pt. II, Item 12 STEP-B, Pt. II, Item 13	Part III, Item 1 Part III, Item 2 Part III, Item 3 Part III, Item 4
D. Functions	SCAT, Pt. IV, Item 15 SCAT, Pt. IV, Item 16 SCAT, Pt. IV, Item 21 SCAT, Pt. IV, Item 24	

Topic IV
Numeration

A. Sets	Author Author Author Author	Part I, Item 22 Part I, Item 23 Part I, Item 24 Part I, Item 25
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Numeration (cont.)

<u>Subtopics</u>	<u>Item Identification</u>	<u>Part and Item Number: Supplementary Test</u>
B. Other Number Bases	CL, Item 36 CL, Item 38 CU, Item 29 CU, Item 37	Part III, Item 15 Part III, Item 22 Part III, Item 14 Part III, Item 13
C. Reading, Writing, Using Numbers	STEP-A, Pt. I, Item 10 STEP-B, Pt. II, Item 14 STEP-B, Pt. II, Item 15 CL, Item 47	Part I, Item 4 Part I, Item 15 Part I, Item 17 Part I, Item 2
D. Roman Numerals	STEP-A, Pt. I, Item 16 Author Author Author	Part I, Item 13 Part I, Item 10 Part I, Item 11 Part I, Item 12

Topic V
Measurement

A. Concepts	SCAT, Pt. IV, Item 12 SCAT, Pt. IV, Item 22 CL, Item 7 Author	Part I, Item 27 Part I, Item 20
B. Conversion of Units	STEP-A, Pt. I, Item 5 STEP-B, Pt. I, Item 17 STEP-B, Pt. II, Item 9 Author	Part I, Item 26 Part I, Item 1 Part III, Item 10 Part III, Item 5
C. Operations- No Conversion	SCAT, Pt. IV, Item 1 SCAT, Pt. IV, Item 2 SCAT, Pt. IV, Item 11 SCAT, Pt. IV, Item 13	
D. Operations-With Conversions	SCAT, Pt. IV, Item 8 SCAT, Pt. IV, Item 18 SCAT, Pt. IV, Item 19 STEP-A, Pt. I, Item 6	

Topic VI
Fundamental Operations with Fractions

A. Concepts	SCAT, Pt. IV, Item 14 CL, Item 32 CU, Item 19 CU, Item 31	Part I, Item 30 Part I, Item 5 Part III, Item 21
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Fundamental Operations with Fractions (cont.)

<u>Subtopics</u>	<u>Item Identification</u>	<u>Part and Item Number: Supplementary Test</u>
B. Addition	SCAT, Pt. II, Item 6 SCAT, Pt. II, Item 10 SCAT, Pt. II, Item 22 SCAT, Pt. IV, Item 20	
C. Subtraction	SCAT, Pt. II, Item 12 SCAT, Pt. II, Item 17 SCAT, Pt. II, Item 25 SCAT, Pt. IV, Item 17	
D. Multiplication	SCAT, Pt. II, Item 15 SCAT, Pt. II, Item 24 SCAT, Pt. IV, Item 23 Author	Part III, Item 18
E. Division	Author CU, Item 20 Author Author	Part III, Item 7 Part III, Item 20 Part III, Item 17 Part III, Item 19

APPENDIX E

SUPPLEMENTARY INTELLECTUAL PROCESSES TESTS

DIRECTIONS FOR EXAMINER

NUMBER COMPARISON TEST

Step 1

(Distribute test books to pupils. Be sure that each pupil has a pencil. A special pencil is not necessary for this test.)

Step 2

(Have pupils fill in their names, etc. on the cover page of the test book.)

Step 3

(Then say:) Read the directions on the cover of your test book silently while I read them aloud.

Test Directions From Pupils' Test Books

In this test, you will work with groups of numbers. Look at the sample below.

Sample

Put an X on the line between the numbers that are not the same

639	_____	639
471430	_____	471530
6538	_____	6537
710	_____	710
432105	_____	432105
432701105	_____	43276105

Look at the two groups of numbers with a line between them. Your job is to compare the numbers in these two groups. When the two groups of numbers are not the same, put an X on the line between them. When the two groups are alike, put no mark on the line. You will have to look very carefully because when the groups of numbers are not the same, usually only one number is changed to cause the two groups to be different.

Look at the first two groups of numbers. Are they alike or are they not the same? They are alike; and so, we put no mark on the line. Look at the second two groups of numbers. Are they alike or are they not the same? They are not the same--471430 is not the same as 471530; 4 is the fourth number in the first group and 5 is the fourth number in the second group; and so, put an X on the line between these two groups of numbers. Practice on the next four examples. Mark the groups of numbers which are not the same.

Number Comparison Test (continued)

Then, we will talk about them. (After pupils have worked examples, check them orally to be sure pupils know what to do, how to mark items, etc.)

When you begin the test on the next page, you will see a note above the double line to remind you that you put an X on the line between the groups of numbers which are not the same.

There are two parts to this test. We will work on Part I first. Then, we will rest. Next, we'll work on Part II. You will have one minute for each part. However, you can't finish a whole part. Nobody could because we have more here than anyone can do in one minute. But do as many as you can.

Work down the page on the left side; then work down the page on the right side if you have time. Work carefully and work as fast as you can.

(End Test Directions From Pupils' Test Books)

Step 4

(Then say:) One more thing. Never turn a page until I tell you to do so. And, each time you turn a page, turn it under your book. Only the page you are working on should show. Never have the back of any page showing. (Throughout the test, watch this carefully and repeat the comment when necessary. This is very important. If all pages are not turned under, pupils can see through the back of the pages. If this happens, the validity of the test might be lessened.)

Are there any questions about what you are to do? (Answer all questions until pupils understand what to do.)

Step 5

(Then say:) Let's do Part I of the test. You have one (1) minute to compare as many groups as you can. Remember, work fast and work accurately. Turn the page and fold it under your test book. Pick up your pencil.

Is everyone ready?

Begin.

Step 6

(Timing is extremely important in this test. Use the stop watch. At the end of exactly one (1) minute say:) Stop! Put your pencils down and rest. Allow the pupils approximately 30 seconds rest between Part I and Part II of the test.)

Number Comparison Test (continued)**Step 7**

(Then say:) Now, let's do Part II (Repeat exactly the directions and procedures in Step 5 and Step 6. After Part II is completed, collect the pupils' tests immediately.)

NUMBER COMPARISON TEST

Put an X on the line between the numbers that are not the same

639 _____ 639	41498 _____ 41598
471430 _____ 471530	6097 _____ 6097
6538 _____ 6537	162539 _____ 162539
432105 _____ 432105	42018591 _____ 43018591
6182653905 _____ 6182653905	6471075 _____ 6471075
43270105 _____ 43276105	721532992 _____ 721582992
27109816 _____ 27109816	3417983 _____ 3417987
51960 _____ 51960	80537051 _____ 80537051
923452170 _____ 923452170	5911306581 _____ 5911306581
3705431 _____ 3105431	836140 _____ 836140
25706652 _____ 25706652	494713 _____ 474713
320185916 _____ 320186916	60826498 _____ 60826478
54710756 _____ 54710756	59305821 _____ 57305821
621532992 _____ 621582992	236031794 _____ 236031294
241798 _____ 241798	8057311 _____ 8051311
70537051 _____ 70537057	48210435 _____ 48210435
736140 _____ 736170	405176841 _____ 405176841
3947130 _____ 3947150	80145349 _____ 80145349
508264987 _____ 508264987	532105 _____ 532105
49305821 _____ 49305821	718265390 _____ 718265390
136031794 _____ 136031794	53270105 _____ 53270105
7057311 _____ 7057361	37109816 _____ 37189816
38210435 _____ 38210535	61960 _____ 61950
710 _____ 710	123452170 _____ 123452190

DO NOT GO ON TO THE NEXT PAGE UNTIL ASKED TO DO SO. STOP.

DIRECTIONS FOR EXAMINER

NUMBER SPAN TEST

Step 1

(Distribute test books to pupils. Be sure that each pupil has a pencil. A special pencil is not necessary for this test.)

Step 2

(Have pupils fill in their names, etc. on the cover page of the test book.)

Step 3

(Then say:) Read the directions on the cover of your test book silently while I read them aloud.

Test Directions From Pupils' Test Books

In this test, you will work with series of single numbers. The numbers will be read aloud to you. After a series is read aloud, write down all of the numbers in exactly the same order in which they are read. Your answers will not be right if you do not put down all of the numbers and put them down in exactly the same order.

Let's do two series for practice. Do not write any number until the whole list has been read. Listen carefully to the numbers. When you hear the word "Begin", write the numbers in the blank space in exactly the same order in which they are read.

(Examiner, note timing instructions in Step 6 below.)

Get your pencils ready.

List A. Ready: 9 7 2 Begin:

List B. Ready: 6 4 1 Begin:

Sample

List A: _____

List B: _____

(After the pupils have done both lists, check the work orally to be sure that the pupils know what to do, how to place the numbers, etc.)

The test has two lists of three numbers, two lists of four numbers, two lists of five numbers, and so on. Some of the lists will be too long for you to remember all of the numbers. If you do not remember some numbers in a list, write as many as you can remember. But, listen carefully, try to remember all of the numbers if possible, and be sure to write them down in exactly the same order they are read.

(End Test Directions From Pupils' Test Books)

Number Span Test (continued)**Step 4**

(Then say:) One more thing. Never turn a page until I tell you to do so. And, each time you turn a page, turn it under your book. Only the page you are working on should show. Never have the back of any page showing. (Throughout the test, watch this carefully and repeat the comment when necessary. This is very important. If all pages are not turned under, pupils can see through the back of the pages. If this happens, the validity of the test might be lessened.)

Are there any questions about what you are to do? (Answer all questions until pupils understand what to do.)

Step 5

(Then say:)

Turn to the next page. Get your pencils ready. The first list will have four numbers in it. Listen very carefully. When you hear the word "Begin", write the numbers exactly as I've said them.

Ready: 1 4 0 6 Begin:

(Continue through the list. Before beginning, note the information presented below.)

Step 6

(Present to the pupils the lists on the attached **LISTS TO BE READ ORALLY BY EXAMINERS**. When the lists are all presented, take up papers immediately.)

Step 7

(**TIMING**. There is no need to use a stop watch. However, read the digits at a rate of approximately one digit per second. Allow approximately two seconds per digit for the pupils to write their responses. Allow approximately three seconds rest between lists after the pupils have finished writing.)

Step 8

(**Caution**. Be sure the pupils keep up; i.e., be sure they are writing the series which you read-- on the correct line on the answer sheet. Help them understand that if they can not write all of a series, they should leave it and go on to the next series as it is read. At the indicated places, remind the pupils to listen carefully, etc.)

NUMBER SPAN TEST

LISTS TO BE READ ORALLY BY EXAMINERS

TIMING: Read at rate of approximately one digit per second.
Allow approximately two seconds per digit for pupils to write their responses. Allow approximately three seconds rest between lists.

Line 1. Ready: 1 4 0 6 Begin:

Line 2. Ready: 2 7 3 0 Begin:

Line 3. Ready: 3 9 4 1 8 Begin:

Line 4. Ready: 8 5 9 4 3 Begin:

REMEMBER. LISTEN CAREFULLY. BE SURE TO WRITE THE NUMBER IN EXACTLY THE SAME ORDER THEY ARE READ.

Line 5. Ready: 0 6 7 2 8 5 Begin:

Line 6. Ready: 7 0 6 2 9 4 Begin:

Line 7. Ready: 3 5 1 6 9 2 7 Begin:

Line 8. Ready: 1 5 3 8 7 9 6 Begin:

REMEMBER. LISTEN CAREFULLY. BE SURE TO WRITE THE NUMBERS IN EXACTLY THE SAME ORDER THEY ARE READ.

Line 9. Ready: 5 8 3 9 1 2 0 4 Begin:

Line 10. Ready: 2 9 0 8 1 3 5 7 Begin:

Line 11. Ready: 7 6 4 5 8 0 1 2 9 Begin:

Line 12. Ready: 0 4 2 8 6 5 1 2 9 Begin:

NUMBER SPAN TEST

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

DIRECTIONS FOR EXAMINER

WORD NUMBER TEST

Step 1

(Distribute test books to pupils. Be sure that each pupil has a pencil. A special pencil is not necessary for this test.)

Step 2

(Have pupils fill in their names, etc. on the cover page of the test book.)

Step 3

(Then say:) Read the directions on the cover of your test book silently while I read them aloud.

Test Directions From Pupils' Test Books

In this test, you will work with pairs of words and numbers. Look at the sample below.

Sample

<u>STUDY LIST</u>		<u>TEST LIST</u>	
<u>WORD</u>	<u>NUMBER</u>	<u>WORD</u>	<u>NUMBER</u>
1. chain	<u>81</u>	1. garage	_____
2. carrot	<u>72</u>	2. locket	_____
3. shoe	<u>37</u>	3. map	_____
4. garage	<u>23</u>	4. carrot	_____
5. map	<u>49</u>		
6. locket	<u>58</u>		
7. telephone	<u>95</u>		
8. baseball	<u>16</u>		

Look at the STUDY LIST of word-number pairs on the left of the double line. These are the pairs of words and numbers which go together to make a pair; chain goes with 81, carrot goes with 72, and so on.

Now look on the right of the double line. This is a part of a TEST LIST. See the column of words. The numbers which go with the words to make word-number pairs are missing. Also, the word-number pairs are in mixed-up order.

Word Number Test (continued)

Your job is to learn words and numbers which go together to make a pair. You do this by reading a STUDY LIST. Then, you will write the missing numbers in a TEST LIST to see how many word-number pairs you have learned.

Let's do the sample. We'll read through the STUDY LIST together; ready: chain 81; carrot 72; shoe 37; garage 23; map 49; locket 58; telephone 95; baseball 16. Now, let's do the test list. After each word is read aloud, write the number which is missing. Then we will check what you have done.

(Examiner, note timing instructions in Step 8 below.)

Get your pencils ready.

1. garage _____
2. locket _____
3. map _____
4. carrot _____

(Examiner: The answers for checking purposes are the following:
garage 23; locket 58; map 49;
carrot 72.)

That's all of the test list we'll do for a sample. When you begin the test, you'll have the whole list of pairs. Now, let's check what you have done. (Check the items orally to be sure that the pupils know what to do, how to place the numbers, etc.)

When you begin the test, you will see a STUDY LIST first. We will read the STUDY LIST together three times. As each word-number pair is read aloud, you will read it silently. Then, we will do the test list. On the test list, after each word is read aloud, you will write the number which goes with the word to make a word-number pair.

One thing to remember. In the study lists and in the test lists, the same words and numbers always go together to make a pair; but, the lists will be in mixed-up order each time. Do not try to learn just a list of numbers because you can't get the right answer that way. You have to learn what words and numbers go together to make a pair.

There are two parts to this test. Part I has a STUDY LIST and a TEST LIST. So does Part II. After we do Part I, we will rest; then we will do Part II. Nobody can learn all of the word-number pairs. The list is too long for that in the time we have to study. But, learn as many word-number pairs as you can. Work carefully and as fast as you can.

(End Test Directions From Pupils' Test Books)

Word Number Test (continued)**Step 4**

(Then say:) One more thing. Never turn a page until I tell you to do so. And, each time you turn a page, turn it under your book. Only the page you are working on should show. Never have the back of any page showing. (Throughout the test, watch this carefully and repeat the comment when necessary. This is very important. If all pages are not turned under, pupils can see through the backs of the pages. If this happens, the validity of the test might be lessened.)

Are there any questions about what you are to do? (Answer all questions until pupils understand what to do.)

Step 5

Turn to the next page, STUDY LIST-1. Read silently with me while I read the word-number pairs aloud. We'll read the list three times. (Read STUDY LIST-1 three times.)

Step 6

Turn to the next page, TEST LIST-1. I'll read the word, you write the number which goes with it. Ready? (Read TEST LIST-1) garage ____; locket ____; etc.

Step 7

(Repeat the procedures in Steps 5 and 6 for STUDY LIST-2 and TEST LIST-2, respectively. After TEST LIST-2 is completed, take up papers immediately.)

Step 8

(Timing. There is no need to use a stop watch. However, read the words and digits at a rate of approximately one word or digit per second. Allow approximately two seconds per digit for the pupils to write their responses. Allow the pupils approximately 30 seconds rest between lists.)

(CAUTION. Be sure the pupils keep up; i.e., be sure they are writing the number after the word you read-- on the correct line on the answer sheet. Help them understand that if they don't know a number which goes with a word, they should leave it alone and go to the next word as it is read.)

WORD NUMBER TEST

STUDY LIST-1

<u>WORD</u>	<u>NUMBER</u>
1. chain	<u>81</u>
2. carrot	<u>72</u>
3. shoe	<u>37</u>
4. garage	<u>23</u>
5. map	<u>49</u>
6. locket	<u>58</u>
7. telephone	<u>95</u>
8. baseball	<u>16</u>

WORD NUMBER TEST

TEST LIST-1

<u>WORD</u>	<u>NUMBER</u>
1. garage	_____
2. locket	_____
3. map	_____
4. carrot	_____
5. baseball	_____
6. telephone	_____
7. shoe	_____
8. chain	_____

WORD NUMBER TEST

STUDY LIST-2

<u>WORD</u>	<u>NUMBER</u>
1. shoe	<u>37</u>
2. map	<u>49</u>
3. locket	<u>58</u>
4. telephone	<u>95</u>
5. chain	<u>81</u>
6. garage	<u>23</u>
7. carrot	<u>72</u>
8. baseball	<u>16</u>

WORD NUMBER TEST

TEST LIST-2

<u>WORD</u>	<u>NUMBER</u>
1. carrot	_____
2. shoe	_____
3. chain	_____
4. baseball	_____
5. garage	_____
6. map	_____
7. telephone	_____
8. locket	_____

DIRECTIONS FOR EXAMINER

EQUATIONS TEST

Step 1

(Distribute test books to pupils. Be sure that each pupil has a pencil. A special pencil is not necessary for this test.)

Step 2

(Have pupils fill in their names, etc. on the cover page of the test book.)

Step 3

(Then say:) Read the directions on the cover of your test book silently while I read them aloud.

TEST DIRECTIONS FROM PUPILS' TEST BOOKS

In this test, you will use symbols and numbers in solving problems. Look at the sample below.

Sample

1	2	3	4	5	6	7	8	9
<	=	X	)	+	#	-	>	(

X and + _____ = and) _____ < and > _____

Look at the boxes above the double line. There are numbers in the top row of the boxes; below each number there is a symbol. Each of these number-symbol pairs go together. Each symbol stands for a number. In other words, the $\leq$ ("less than" sign) goes with 1; the $=$ ("equals" sign) goes with 2, and so on.

Look below the double line. There is a row of equations. Here you see combinations of symbols with the answers missing. Your job is to write in the blank space the missing answer to each combination or equation. Look at the first equation. It's X ("times" sign) and + ("plus" sign). What answer goes with X and +? To find out, look at the boxes at the top of the page. The number which goes with X is 3. The number which goes with + is 5. 3 and 5 are 8. The $\geq$ ("greater than" sign) is the symbol for 8. Therefore, $\geq$ is the answer in the equation X and +. Write it in the answer space. Try the other two equations for practice; then, we will talk about them. (After pupils have worked the examples, check them orally to be sure pupils know what to do, how to mark items, etc.)

Equations test (continued)

When you are working the problems, you may look up at the boxes at the top of the page as often as you like to see which number goes with each symbol.

There are two parts to this test. We will work on Part I first. Then we will rest. Next, we'll work on Part II. You will have one minute to work on each part. However, you can't finish a whole part. Nobody could because we have more here than anyone can do in one minute. But do as many as you can.

Work across the page, not down the page. Work carefully and work as fast as you can.

(End Test Directions From Pupils' Test Books.)

Step 4

(Then say:) One more thing. Never turn a page until I tell you to do so. And, each time you turn a page, turn it under your book. Only the page you are working on should show. Never have the back of any page showing. (Throughout the test, watch this carefully and repeat the comment when necessary. This is very important. If all pages are not turned under, pupils can see through the backs of the pages. If this happens, the validity of the test might be lessened.)

Are there any questions about what you are to do? (Answer all questions until pupils understand what to do.)

Step 5

(Then say:) Let's do Part I of the test. You have one (1) minute to do as many equations as you can. Remember, work fast and work accurately. Turn the page and fold it under your test book. Pick up your pencil.

Is everyone ready?

Begin.

Step 6

(Timing is extremely important in this test. Use the stop watch. At the end of exactly one (1) minute say:) Stop! Put your pencils down and rest. Allow the pupils approximately 30 seconds rest between Part I and Part II of the test.)

Step 7

(Then say:) Now, let's do Part II (Repeat exactly the directions and procedures in Step 5 and Step 6. After Part II is completed, collect the pupils' tests immediately.)

EQUATIONS TEST

1	2	3	4	5	6	7	8	9
<	=	X	)	+	#	-	>	(

X and + _____	= and) _____	< and < _____	= and + _____
+ and) _____	X and = _____	X and + _____	= and) _____
= and < _____	= and) _____	+ and) _____	= and < _____
+ and) _____	X and + _____	X and = _____	< and X _____
= and < _____	) and X _____	< and < _____	X and + _____
X and = _____	= and < _____	< and X _____	X and + _____
< and X _____	+ and) _____	= and) _____	= and < _____
= and) _____	) and X _____	< and < _____	< and X _____
+ and) _____	X and = _____	X and + _____	< and < _____
+ and) _____	= and) _____	< and < _____	X and + _____
X and = _____	= and < _____	< and X _____	+ and = _____
< and X _____	X and + _____	< and < _____	= and) _____
) and X _____	+ and) _____	) and X _____	< and X _____
) and = _____	X and = _____	= and + _____	= and < _____
+ and) _____	< and < _____	and X _____	= and # _____
X and = _____	X and + _____	= and) _____	< and X _____
+ and) _____	= and) _____	< and X _____	< and < _____

STOP! DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

DIRECTIONS FOR EXAMINER

NUMBER SERIES TEST

Step 1

(Distribute test books to pupils. Be sure that each pupil has a pencil. A special pencil is not necessary for this test.)

Step 2

(Have pupils fill in their names, etc. on the cover page of the test books.)

Step 3

(Then say:) Read the directions on the cover of your test book silently while I read them aloud.

Test Directions From Pupils' Test Books

In this test, you will be working with series of eight numbers which have been arranged according to a rule. Look at the sample below.

Sample

1.	6	7	8	9	10	11	_____	_____
2.	35	34	33	32	31	30	_____	_____

Look at the series of numbers in the first row of the sample. Six numbers in the series are given to you; the blank spaces mean that two numbers in the series are missing. The numbers you see are related in a certain way according to a rule. Your job is to figure out the rule which shows how these numbers are related. Then, you are to use the rule to find the next two numbers in the series, the missing numbers. You will write the missing numbers in the blank spaces.

In the first row, the numbers are related according to this rule: the numbers increase by one; each number is one more than the number which comes before it. Look at the series: 6 7 8 9 10 11. What are the next two numbers? That's right--12 and 13: $11 + 1 = 12$; and $12 + 1 = 13$. For practice, look at the next row. Find the rule that tells how the numbers are related; then, use the rule to supply the missing numbers. Then we will check your answer. (Check items orally to be sure that the pupils know what to do, how to place the number, etc.)

The test is made up of several series of numbers. A new series appears in each row; and so, a different rule is needed for each row. Do not try to use on a new row, a rule you have used before. It will not work. For each row, figure out the new rule; then supply the missing number. If you can't figure out the rule for a row, go on to the next row. Do the best you can with each row, but move on quickly if you can't find the rule. In other rows, there will be rules which you can find,

Number Series Test (continued)

There are two parts to the test. We will work on Part I first. Then, we will rest. Next, we will work on Part II. You will have 5 minutes for each part. You won't be able to finish either part. Nobody could because there is more here than anyone can do in five minutes. But do as many as you can.

(End Test Directions From Pupils' Test Books)**Step 4**

(Then say:) One more thing. Never turn a page until I tell you to do so. And, each time you turn a page, turn it under your book. Only the page you are working on should show. Never have the back of any page showing. (Throughout the test, watch this carefully and repeat the comment when necessary. This is very important. If all pages are not turned under, pupils can see through the backs of the pages. If this happens, the validity of the test might be lessened.)

Are there any questions about what you are to do? (Answer all questions until pupils understand what to do.)

Step 5

(Then say:) Let's do Part I of the test. You have five (5) minutes to find and use as many rules as you can. Remember, work fast and work accurately. Turn the page and fold it under your test book. Pick up your pencil.

Is everyone ready?

Begin.

Step 6

(Timing is extremely important in this test. Use the stop watch. At the end of exactly five (5) minutes say:) Stop! Put your pencils down and rest. Allow the pupils approximately two (2) minutes rest between Part I and Part II of the test.)

Step 7

(Then say:) Now, let's do Part II. (Repeat exactly the directions and procedures in Step 5 and Step 6. After Part II is completed, collect the pupils' tests immediately.)

NUMBER SERIES TEST

1.	6	7	8	9	10	11	_____	_____
2.	35	34	33	32	31	30	_____	_____
3.	40	45	50	55	60	65	_____	_____
4.	5	5	6	6	7	7	_____	_____
5.	22	24	26	28	30	32	_____	_____
6.	65	60	55	50	45	40	_____	_____
7.	37	47	57	67	77	87	_____	_____
8.	18	21	24	27	30	33	_____	_____
9.	48	46	44	42	40	38	_____	_____
10.	34	45	56	67	78	89	_____	_____
11.	8	8	6	6	4	4	_____	_____
12.	3	7	11	15	19	23	_____	_____
13.	9	1	7	1	5	1	_____	_____
14.	25	25	21	21	17	17	_____	_____
15.	4	5	8	9	12	13	_____	_____
16.	21	18	16	13	11	8	_____	_____
17.	1	2	4	8	16	32	_____	_____
18.	3	4	6	9	13	18	_____	_____
19.	12	14	13	15	14	16	_____	_____
20.	25	24	22	21	19	18	_____	_____
21.	16	12	15	11	14	10	_____	_____
22.	15	16	14	17	13	18	_____	_____
23.	1	4	9	16	25	36	_____	_____
24.	21	18	16	15	12	10	_____	_____
25.	4	8	10	20	22	44	_____	_____

STOP! DO NOT TURN THE PAGE UNTIL TOLD TO DO SO.

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APPENDIX F

PROJECT INFORMATION GUIDE FOR TEACHERS

MATHEMATICS PROJECT**Introduction**

The mathematics project is being conducted under the joint sponsorship of the Clarke County School District, the College of Education at the University of Georgia, and the Cooperative Research Branch of the U. S. Office of Education. The general purposes are to obtain information about how fourth, fifth, and sixth grade pupils achieve in mathematics and the relationship of certain intellectual processes to mathematics achievement. This information will be used to provide a partial basis for developing new procedures in mathematics education.

Schedule

Chart I contains information about the names of the tests, the dates and sessions they are to be administered, and the specific timing for each test.

Instructions about timing also appear in the examiner's directions for each test. Use stop watches where indicated. Timing instructions should be carefully observed. Deviations could lessen the validity of the tests.

Note the subtests on the SCAT and the PMA tests which are not to be administered to the pupils. These include the following: SCAT--Part I and Part III; PMA--Number Sense and Addition Test.

Directions for Tests

The SCAT test, the Mathematics Test Supplement, and the PMA tests involve IBM answer sheets. Some pupils, especially fourth graders, have not had much experience with this type of answer sheet. One illustration of how to use the sheet is presented with each test. However, additional illustrations on the blackboard may be necessary for some pupils.

A set of directions is provided for each test. Please do not vary from these directions but feel free to repeat them as often as necessary; also, when indicated, help pupils with words they cannot read. However, exercise care in order to avoid giving pupils any interpretations of test materials.

In the several sets of directions, comments are made (1) to reassure the pupils that they will not know the answers to all questions and (2) to caution them not to spend too much time on questions which they do not know. Generally, wherever indicated, continue to reassure the pupils and to keep them moving along on the tests.

**Special Instructions for Answer Sheets:
SCAT and Mathematics Test Supplements**

The two sections of the Mathematics Test Supplements will be substituted for Part I and Part III of the SCAT. These two sections of the

supplements have been designed to correspond with Part I and Part III of the SCAT answer sheet.

The two SCAT mathematics sections (Part II and Part IV) should be administered during the first session on May 8; the two sections of the Mathematics Test Supplement (Part I and Part III) should be administered during the second session on May 8.

Be sure that the pupils put their answers in the appropriate sections of the answer sheets. The sections of the answer sheets used must correspond with the labels of the part of the mathematics tests being given. Specifically,

- 1) for Part II, SCAT -- begin top right section of answer sheet;
- 2) for Part IV, SCAT -- begin bottom right section of answer sheet;
- 3) for Part I, Math. Test Supplement -- begin top left section of answer sheet;
- 4) for Part III, Math. Test Supplement -- begin bottom left section of answer sheet.

In addition, the pupils need only to write in their names on the answer sheets. Other information called for on the top of the answer sheet will be obtained from the Pupil Information Sheet.

Make-up Tests

No make-up tests will be given.

Delivery and Return of Materials

Tests and testing supplies (stop watches, IBM pencils and answer sheets) will be delivered to the principal's office before school begins on the scheduled test date: mathematics materials will be delivered on the morning of May 8; primary mental abilities tests, on the morning of May 12.

A box of materials will be prepared for each teacher on the basis of her class enrollment. Extra materials will be left with the principals for use if emergencies arise.

Materials will be collected on the afternoon of the respective scheduled test dates. Please turn in the boxes of materials to the principals. Please prepare the materials for return in the following manner.

- 1) Repack materials in the boxes in which they were delivered. Refer to the Materials Check Sheet (example included on next page) to find dates for delivery and return of specific materials. A copy of the Materials Check Sheet marked by the project staff will be in each box of materials. Mark the appropriate places as you repack the materials. Return this Materials Check Sheet in the box on each testing day.
- 2) The Pupil Information Sheet will be delivered on May 8. Complete this during the interval between May 8 and May 12. Repack it in the box on May 12.

CHART I: Schedule

Date/ Session	Name of Test Battery	Timing of Tests	Total Time*
5/8 First Session	SCAT (ETS)	Part II: 20 minutes Part IV: 25 minutes	45 min.
REST BREAK			15-30 min.
5/8 Second Session	Mathematics Test Supplement (ETS, CTB)	Part I: 25 minutes Part III: 30 minutes	55 min.
5/12 First Session	PMA Tests (SRA)	Word Test: 7 minutes Spatial Relations Test: 6 minutes Word Grouping Test: 6 minutes	19 min.
REST BREAK			15-30 min.
5/12 Second Session	Supplementary Tests	Digit Symbol Equations: 60 seconds each part Number Comparisons: 60 seconds each part Number Span: 1 second per unit for examiner presentation; 2 seconds per unit for pupil responses. Word Number: Same timing as for Number Span Test Number Series Test: 5 minutes each part	Approx. 20 min.

* Not including time for directions.

MATERIALS CHECK SHEET--MATHEMATICS PROJECT

Teacher's Name _____ School _____

Item	Number Delivered	Numbered Returned
Testing Session May 8		
A. SCAT: Tests and answer sheets	_____	_____
B. Math. Supplement: Test and answer sheets	_____	_____
C. Extra tests and answer sheets	Principal's Office	-----
D. IBM Pencils	_____	Keep until 5/12
E. Pupil Information Sheet	_____	Keep until 5/12

Testing Sessions: May 12

A. PMA Test: Tests and answer sheets	_____	_____
B. Supplementary Intellectual Processes Tests	_____	_____
1. Digit Symbol Equations	_____	_____
2. Number Comparison	_____	_____
3. Number Span	_____	_____
4. Word-Number	_____	_____
5. Number Series	_____	_____
C. Extra Tests	(Principal's Office)	-----
D. IBM pencils	---- on 5/8	_____
E. Stop watches	_____	_____
F. Examiner's manual (notebook)	_____	_____
G. Pupil Information Sheet	1 on May 8	_____

PUPIL INFORMATION SHEET

Mathematics Project

Teacher's

Name _____

School _____

Grade _____

Please list below in alphabetical order, with last names first, the names and birthdates of the pupils in your class. Indicate with an asterisk (*) the names of any pupils whose test results should be excluded from the analyses. Pupils whose results are to be excluded would be those who in your judgment have either physical or emotional difficulties which might interfere with their responses to the tests.

Name	Birthdate	Name	Birthdate
1. _____	_____	21. _____	_____
2. _____	_____	22. _____	_____
3. _____	_____	23. _____	_____
4. _____	_____	24. _____	_____
5. _____	_____	25. _____	_____
6. _____	_____	26. _____	_____
7. _____	_____	27. _____	_____
8. _____	_____	28. _____	_____
9. _____	_____	29. _____	_____
10. _____	_____	30. _____	_____
11. _____	_____	31. _____	_____
12. _____	_____	32. _____	_____
13. _____	_____	33. _____	_____
14. _____	_____	34. _____	_____
15. _____	_____	35. _____	_____
16. _____	_____	36. _____	_____
17. _____	_____	37. _____	_____
18. _____	_____	38. _____	_____
19. _____	_____	39. _____	_____
20. _____	_____	40. _____	_____

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APPENDIX G

**SIMPLE CORRELATIONS BETWEEN MATHEMATICS VARIABLES
AND INTELLECTUAL PROCESSES VARIABLES:
GRADES FOUR, FIVE, AND SIX**

**Simple Correlations Between Mathematics Variables and
Intellectual Processes Variables: Grades 4, 5, 6**

Mathematics Variable Y_i	Intellectual Processes Variable X_i	Simple Correlation Coefficient		
		Grade 4 n = 244	Grade 5 n = 261	Grade 6 n = 240
Y1 Addition	X1 Perceptual Discrimination -- Sets of Digits	.178	.084	.065
	X2 Memory Span	.016	-.092	-.031
	X3 Associative Memory	.200	.077	.097
	X4 Associative Memory Plus Conversion	.162	.307	.168
	X5 Reasoning -- Number Relations	.171	.207	.098
	X6 Verbal Meaning -- Synonyms	.144	.171	.035
	X7 Spatial Relations -- Visualization	.013	.206	-.016
	X8 Reasoning -- Word Grouping	.172	.222	.079
Y2 Subtraction	X1 Perceptual Discrimination -- Sets of Digits	.190	.095	.236
	X2 Memory Span	.011	.061	-.022
	X3 Associative Memory	.254	.197	.237
	X4 Associative Memory Plus Conversion	.269	.272	.221
	X5 Reasoning -- Number Relations	.434	.469	.307
	X6 Verbal Meaning -- Synonyms	.397	.427	.403
	X7 Spatial Relations -- Visualization	.148	.319	.138
	X8 Reasoning -- Word Grouping	.381	.417	.343

Mathematics Variable Y_1	Intellectual Processes Variable X_1	Simple Correlation Coefficient		
		Grade 4 n = 244	Grade 5 n = 261	Grade 6 n = 240
Y3 Multiplication	X1 Perceptual Discrimination -- Sets of Digits	.130	.097	.295
	X2 Memory Span	-.055	-.007	.005
	X3 Associative Memory	.152	.281	.428
	X4 Associative Memory Plus Conversion	.171	.381	.370
	X5 Reasoning -- Number Relations	.449	.550	.487
	X6 Verbal Meaning -- Synonyms	.422	.439	.477
	X7 Spatial Relations -- Visualization	.227	.280	.247
	X8 Reasoning -- Word Grouping	.317	.501	.466
Y4 Division	X1 Perceptual Discrimination -- Sets of Digits	.083	.063	.328
	X2 Memory Span	.068	-.076	.095
	X3 Associative Memory	.072	.275	.289
	X4 Associative Memory Plus Conversion	.084	.319	.392
	X5 Reasoning -- Number Relations	.380	.515	.325
	X6 Verbal Meaning -- Synonyms	.259	.401	.388
	X7 Spatial Relations -- Visualization	.270	.318	.238
	X8 Reasoning -- Word Grouping	.265	.454	.364

Mathematics Variable Y_1	Intellectual Processes Variable X_1	Simple Correlation Coefficient		
		Grade 4 n = 244	Grade 5 n = 261	Grade 6 n = 240
Y5 Laws and Generalizations	X1 Perceptual Discrimination -- Sets of Digits	.124	.007	.081
	X2 Memory Span	.082	-.047	-.014
	X3 Associative Memory	.200	.217	.173
	X4 Associative Memory Plus Conversion	.223	.252	.166
	X5 Reasoning -- Number Relations	.459	.412	.319
	X6 Verbal Meaning -- Synonyms	.437	.395	.287
	X7 Spatial Relations -- Visualization	.197	.279	.149
	X8 Reasoning -- Word Grouping	.283	.397	.254
Total Topic: Number Operations, Assumptions	X1 Perceptual Discrimination -- Sets of Digits	.214	.109	.327
	X2 Memory Span	.031	-.046	.018
	X3 Associative Memory	.267	.328	.393
	X4 Associative Memory Plus Conversion	.278	.467	.426
	X5 Reasoning -- Number Relations	.583	.665	.484
	X6 Verbal Meaning -- Synonyms	.514	.560	.503
	X7 Spatial Relations -- Visualization	.264	.423	.245
	X8 Reasoning -- Word Grouping	.436	.612	.478

Mathematics Variable Y_i	Intellectual Processes Variable X_i	Simple Correlation Coefficient		
		Grade 4 n = 244	Grade 5 n = 261	Grade 6 n = 240
Y7 Definition of Terms	X1 Perceptual Discrimination -- Sets of Digits	.148	-.127	.057
	X2 Memory Span	-.056	-.139	.053
	X3 Associative Memory	.017	.117	.162
	X4 Associative Memory Plus Conversion	-.044	.154	.284
	X5 Reasoning -- Number Relations	.117	.280	.328
	X6 Verbal Meaning -- Synonyms	.112	.230	.301
	X7 Spatial Relations -- Visualization	.021	.186	.196
	X8 Reasoning -- Word Grouping	.052	.196	.295
Y8 Recognition of Geometric Figures	X1 Perceptual Discrimination -- Sets of Digits	.124	.009	.087
	X2 Memory Span	-.025	-.090	-.054
	X3 Associative Memory	.155	.162	.268
	X4 Associative Memory Plus Conversion	.178	.291	.331
	X5 Reasoning -- Number Relations	.299	.451	.379
	X6 Verbal Meaning -- Synonyms	.349	.425	.516
	X7 Spatial Relations -- Visualization	.142	.348	.312
	X8 Reasoning -- Word Grouping	.287	.423	.455

Mathematics Variable Y_1	Intellectual Processes Variable X_1	Simple Correlation Coefficient		
		Grade 4 n = 244	Grade 5 n = 261	Grade 6 n = 240
Y9 Measurement of Geometric Figures	X1 Perceptual Discrimination -- Sets of Digits	.073	.120	.249
	X2 Memory Span	.122	.102	.063
	X3 Associative Memory	.138	.182	.303
	X4 Associative Memory Plus Conversion	.113	.298	.418
	X5 Reasoning -- Number Relations	.344	.476	.539
	X6 Verbal Meaning -- Synonyms	.364	.401	.525
	X7 Spatial Relations -- Visualization	.245	.311	.350
	X8 Reasoning -- Word Grouping	.255	.452	.540
Y10 Total Topic: Geometry	X1 Perceptual Discrimination -- Sets of Digits	.170	.008	.181
	X2 Memory Span	.016	-.049	.032
	X3 Associative Memory	.151	.208	.326
	X4 Associative Memory Plus Conversion	.121	.337	.460
	X5 Reasoning -- Number Relations	.368	.548	.558
	X6 Verbal Meaning -- Synonyms	.400	.478	.594
	X7 Spatial Relations -- Visualization	.195	.383	.381
	X8 Reasoning -- Word Grouping	.289	.488	.575

Mathematics Variable Y_i	Intellectual Processes Variable X_i	Simple Correlation Coefficient		
		Grade 4 $n = 244$	Grade 5 $n = 261$	Grade 6 $n = 240$
Y11 Equality	X1 Perceptual Discrimination -- Sets of Digits	.173	-.028	.139
	X2 Memory Span	.167	-.047	-.057
	X3 Associative Memory	.255	.238	.247
	X4 Associative Memory Plus Conversion	.317	.261	.295
	X5 Reasoning -- Number Relations	.522	.520	.463
	X6 Verbal Meaning -- Synonyms	.495	.458	.374
	X7 Spatial Relations -- Visualization	.235	.278	.295
	X8 Reasoning -- Word Grouping	.406	.464	.415
Y12 Order Relations	X1 Perceptual Discrimination -- Sets of Digits	.165	.091	.155
	X2 Memory Span	.154	-.016	-.009
	X3 Associative Memory	.132	.255	.276
	X4 Associative Memory Plus Conversion	.189	.320	.364
	X5 Reasoning -- Number Relations	.230	.559	.535
	X6 Verbal Meaning -- Synonyms	.339	.552	.545
	X7 Spatial Relations -- Visualization	.179	.336	.298
	X8 Reasoning -- Word Grouping	.298	.489	.517

Mathematics Variable Y_i	Intellectual Processes Variable X_i	Simple Correlation Coefficient		
		Grade 4 n = 244	Grade 5 n = 261	Grade 6 n = 240
Y13 Maps, Graphs, Charts	X1 Perceptual Discrimination -- Sets of Digits	.006	.059	.168
	X2 Memory Span	.045	-.001	-.000
	X3 Associative Memory	.130	.219	.223
	X4 Associative Memory Plus Conversion	.186	.252	.199
	X5 Reasoning -- Number Relations	.310	.351	.322
	X6 Verbal Meaning -- Synonyms	.333	.410	.330
	X7 Spatial Relations -- Visualization	.115	.255	.198
	X8 Reasoning -- Word Grouping	.171	.395	.275
Y14 Functions	X1 Perceptual Discrimination -- Sets of Digits	.161	.132	.297
	X2 Memory Span	.006	.135	.068
	X3 Associative Memory	.227	.262	.284
	X4 Associative Memory Plus Conversion	.120	.331	.294
	X5 Reasoning -- Number Relations	.368	.507	.425
	X6 Verbal Meaning -- Synonyms	.387	.392	.423
	X7 Spatial Relations -- Visualization	.167	.325	.278
	X8 Reasoning -- Word Grouping	.299	.505	.493

Mathematics Variable Y_i	Intellectual Processes Variable X_i	Simple Correlation Coefficient		
		Grade 4 n = 244	Grade 5 n = 261	Grade 6 n = 240
Y15 Total Topic Relations	X1 Perceptual Discrimination -- Sets of Digits	.187	.094	.268
	X2 Memory Span	.135	.030	.005
	X3 Associative Memory	.277	.347	.361
	X4 Associative Memory Plus Conversion	.299	.417	.408
	X5 Reasoning -- Number Relations	.530	.693	.614
	X6 Verbal Meaning -- Synonyms	.577	.645	.593
	X7 Spatial Relations -- Visualization	.258	.427	.374
	X8 Reasoning -- Word Grouping	.434	.661	.604
Y16 Sets	X1 Perceptual Discrimination -- Sets of Digits	.288	.086	.164
	X2 Memory Span	.127	.009	-.016
	X3 Associative Memory	.161	.189	.095
	X4 Associative Memory Plus Conversion	.316	.222	.137
	X5 Reasoning -- Number Relations	.469	.299	.256
	X6 Verbal Meaning -- Synonyms	.429	.317	.223
	X7 Spatial Relations -- Visualization	.277	.264	.191
	X8 Reasoning -- Word Grouping	.417	.296	.207

Mathematics Variable Y_1	Intellectual Processes Variable X_1	Simple Correlation Coefficient		
		Grade 4 $n = 244$	Grade 5 $n = 261$	Grade 6 $n = 240$
Y17 Number Bases	X1 Perceptual Discrimination -- Sets of Digits	.253	.018	.136
	X2 Memory Span	.098	.009	.035
	X3 Associative Memory	.099	.176	.168
	X4 Associative Memory Plus Conversion	.328	.233	.253
	X5 Reasoning -- Number Relations	.297	.508	.355
	X6 Verbal Meaning -- Synonyms	.393	.423	.336
	X7 Spatial Relations -- Visualization	.148	.279	.231
	X8 Reasoning -- Word Grouping	.395	.435	.358
Y18 Reading, Writing, Using Numbers	X1 Perceptual Discrimination -- Sets of Digits	.231	.210	.200
	X2 Memory Span	.066	.054	.036
	X3 Associative Memory	.232	.242	.330
	X4 Associative Memory Plus Conversion	.336	.272	.330
	X5 Reasoning -- Number Relations	.561	.544	.525
	X6 Verbal Meaning -- Synonyms	.505	.540	.547
	X7 Spatial Relations -- Visualization	.284	.343	.288
	X8 Reasoning -- Word Grouping	.448	.523	.550

Mathematics Variable Y_i	Intellectual Processes Variable X_i	Simple Correlation Coefficient		
		Grade 4 n = 244	Grade 5 n = 261	Grade 6 n = 240
Y19 Roman Numerals	X1 Perceptual Discrimination -- Visualization	.169	.040	.120
	X2 Memory Span	.164	.088	.090
	X3 Associative Memory	.228	.243	.354
	X4 Associative Memory Plus Conversion	.314	.239	.266
	X5 Reasoning -- Number Relations	.445	.505	.044
	X6 Verbal Meaning -- Synonyms	.444	.499	.412
	X7 Spatial Relations -- Visualization	.277	.232	.282
	X8 Reasoning -- Word Grouping	.326	.460	.466
Y20 Total: Numeration	X1 Perceptual Discrimination -- Sets of Digits	.326	.115	.228
	X2 Memory Span	.158	.056	.047
	X3 Associative Memory	.245	.295	.355
	X4 Associative Memory Plus Conversion	.449	.336	.378
	X5 Reasoning -- Number Relations	.607	.654	.590
	X6 Verbal Meaning -- Synonyms	.613	.622	.567
	X7 Spatial Relations -- Visualization	.339	.387	.371
	X8 Reasoning -- Word Grouping	.549	.600	.593

Mathematics Variable Y_i	Intellectual Processes Variable X_i	Simple Correlation Coefficient		
		Grade 4 $n = 244$	Grade 5 $n = 261$	Grade 6 $n = 240$
Y21 Concepts of Measurement	X1 Perceptual Discrimination -- Sets of Digits	.140	.133	.229
	X2 Memory Span	.180	.063	.097
	X3 Associative Memory	.215	.270	.314
	X4 Associative Memory Plus Conversion	.278	.300	.295
	X5 Reasoning -- Number Relations	.373	.454	.411
	X6 Verbal Meaning -- Synonyms	.368	.440	.502
	X7 Spatial Relations -- Visualization	.209	.364	.299
	X8 Reasoning -- Word Grouping	.320	.459	.487
Y22 Conversion of Units	X1 Perceptual Discrimination -- Sets of Digits	.103	.074	.218
	X2 Memory Span	.005	.063	.075
	X3 Associative Memory	.205	.183	.395
	X4 Associative Memory Plus Conversion	.178	.306	.379
	X5 Reasoning -- Number Relations	.476	.546	.514
	X6 Verbal Meaning -- Synonyms	.436	.517	.498
	X7 Spatial Relations -- Visualization	.245	.368	.278
	X8 Reasoning -- Word Grouping	.399	.492	.466

Mathematics Variable Y_1	Intellectual Processes Variable X_1	Simple Correlation Coefficient		
		Grade 4 n = 244	Grade 5 n = 261	Grade 6 n = 240
Y23 Operations - No Conversion of units	X1 Perceptual Discrimination -- Sets of Digits	.231	.138	.273
	X2 Memory Span	.179	-.057	-.081
	X3 Associative Memory	.255	.211	.227
	X4 Associative Memory Plus Conversion	.416	.301	.244
	X5 Reasoning -- Number Relations	.515	.500	.405
	X6 Verbal Meaning -- Synonyms	.504	.507	.377
	X7 Spatial Relations -- Visualization	.306	.350	.237
	X8 Reasoning -- Word Grouping	.483	.527	.381
Y24 Operations - Conversion of units	X1 Perceptual Discrimination -- Sets of Digits	.101	.039	.247
	X2 Memory Span	.150	.012	.020
	X3 Associative Memory	.128	.187	.335
	X4 Associative Memory Plus Conversion	.210	.267	.349
	X5 Reasoning -- Number Relations	.390	.544	.506
	X6 Verbal Meaning -- Synonyms	.445	.363	.513
	X7 Spatial Relations -- Visualization	.160	.382	.305
	X8 Reasoning -- Word Grouping	.275	.387	.495

Mathematics Variable Y_i	Intellectual Processes Variable X_i	Simple Correlation Coefficient		
		Grade 4 $n = 244$	Grade 5 $n = 260$	Grade 6 $n = 240$
Y25 Total: Measurement	X1 Perceptual Discrimination -- Sets of Digits	.197	.126	.302
	X2 Memory Span	.180	.030	.048
	X3 Associative Memory	.276	.284	.411
	X4 Associative Memory Plus Conversion	.372	.389	.407
	X5 Reasoning -- Number Relations	.601	.678	.584
	X6 Verbal Meaning -- Synonyms	.601	.603	.607
	X7 Spatial Relations -- Visualization	.315	.487	.359
	X8 Reasoning -- Word Grouping	.505	.616	.586
Y26 Concepts of Fractions	X1 Perceptual Discrimination -- Sets of Digits	.249	.089	.245
	X2 Memory Span	.242	-.124	-.085
	X3 Associative Memory	.201	.103	.336
	X4 Associative Memory Plus Conversion	.234	.248	.378
	X5 Reasoning -- Number Relations	.270	.424	.452
	X6 Verbal Meaning -- Synonyms	.349	.315	.538
	X7 Spatial Relations -- Visualization	.214	.262	.338
	X8 Reasoning -- Word Grouping	.281	.291	.547

Mathematics Variable Y_i	Intellectual Processes Variable X_i	Simple Correlation Coefficient		
		Grade 4 $n = 244$	Grade 5 $n = 261$	Grade 6 $n = 240$
Y27 Addition of Fractions	X1 Perceptual Discrimination -- Sets of Digits	.176	.145	.217
	X2 Memory Span	.151	.045	.014
	X3 Associative Memory	.224	.254	.347
	X4 Associative Memory Plus Conversion	.247	.349	.345
	X5 Reasoning -- Number Relations	.420	.594	.452
	X6 Verbal Meaning -- Synonyms	.477	.458	.466
	X7 Spatial Relations -- Visualization	.241	.369	.285
	X8 Reasoning -- Word Grouping	.327	.560	.547
Y28 Subtraction of Fractions	X1 Perceptual Discrimination -- Sets of Digits	.210	.088	.247
	X2 Memory Span	.147	-.074	-.009
	X3 Associative Memory	.240	.303	.343
	X4 Associative Memory Plus Conversion	.394	.398	.384
	X5 Reasoning -- Number Relations	.442	.623	.452
	X6 Verbal Meaning -- Synonyms	.457	.505	.538
	X7 Spatial Relations -- Visualization	.213	.475	.231
	X8 Reasoning -- Word Grouping	.389	.544	.436

Mathematics Variable Y_1	Intellectual Processes Variable X_1	Simple Correlation Coefficient		
		Grade 4 $n = 244$	Grade 5 $n = 261$	Grade 6 $n = 240$
Y29 Multiplication of Fractions	X1 Perceptual Discrimination -- Sets of Digits	.077	.156	.286
	X2 Memory Span	.069	.063	.042
	X3 Associative Memory	-.000	.167	.357
	X4 Associative Memory Plus Conversion	.057	.229	.429
	X5 Reasoning -- Number Relations	.145	.363	.418
	X6 Verbal Meaning -- Synonyms	.144	.362	.541
	X7 Spatial Relations -- Visualization	.171	.227	.339
	X8 Reasoning -- Word Grouping	.077	.333	.532
Y30 Division of Fractions	X1 Perceptual Discrimination -- Sets of Digits	.035	.085	.070
	X2 Memory Span	.108	-.057	.074
	X3 Associative Memory	.136	.095	.231
	X4 Associative Memory Plus Conversion	.170	.142	.347
	X5 Reasoning -- Number Relations	.106	.149	.323
	X6 Verbal Meaning -- Synonyms	.114	.044	.420
	X7 Spatial Relations -- Visualization	.066	.222	.234
	X8 Reasoning -- Word Grouping	.108	.108	.329

Mathematics Variable Y_i	Intellectual Processes Variable X_i	Simple Correlation Coefficient		
		Grade 4 $n = 244$	Grade 5 $n = 261$	Grade 6 $n = 240$
Y31 Operations with Fractions	X1 Perceptual Discrimination -- Sets of Digits	.256	.172	.282
	X2 Memory Span	.245	-.047	.012
	X3 Associative Memory	.279	.289	.428
	X4 Associative Memory Plus Conversion	.381	.428	.502
	X5 Reasoning -- Number Relations	.467	.677	.556
	X6 Verbal Meaning -- Synonyms	.523	.528	.666
	X7 Spatial Relations -- Visualization	.302	.486	.379
	X8 Reasoning -- Word Grouping	.406	.577	.633

APPENDIX H
RESULTS OF THE STEP-WISE REGRESSION ANALYSES:
GRADES FOUR, FIVE, AND SIX

Results of the Step-wise Regression Analysis: Grade 4

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y1 Addition	X1 Perceptual Discrimination					
	-- Sets of Digits	0.156	0.035	0.014	2.480	<.05
	X3 Associative Memory	0.181	0.039	0.014	2.877	<.01
	Y Intercept = 2.367					
	R = 0.252; R^2 (in %) = 6.38					
Y2 Subtraction	Partial r for Y1 X1 = 0.158; $t = 2.480$; P <.05					
	Partial r for Y1 X3 = 0.182; $t = 2.877$; P <.01					
	X3 Associative Memory	0.136	0.031	0.013	2.313	<.05
	X5 Reasoning -- Number Relations	0.276	0.033	0.008	3.914	<.001
	X6 Verbal Meaning -- Synonyms	0.208	0.034	0.011	2.990	<.01
Y2 Subtraction	Y Intercept = 1.526					
	R = 0.487; R^2 (in %) = 23.69					
	Partial r for Y2 X3 = 0.148; $t = 2.313$; P <.05					
	Partial r for Y2 X5 = 0.245; $t = 3.914$; P <.001					
	Partial r for Y2 X6 = 0.190; $t = 2.990$; P <.01					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y3 Multiplication	X5 Reasoning -- Number Relations	0.308	0.044	0.010	4.459	<.001
	X6 Verbal Meaning -- Synonyms	0.242	0.048	0.014	3.506	<.001
	Y intercept = 0.109 $R = 0.490$; R^2 (in %) = 24.04 Partial r for Y3 X5 = 0.276; $t = 4.459$; $P < .001$ Partial r for Y3 X6 = 0.220; $t = 3.506$; $P < .001$					
Y4 Division	X5 Reasoning -- Number Relations	0.324	0.036	0.007	4.999	<.001
	X7 Spatial Relations -- Visualization	0.136	0.032	0.015	2.098	<.05
	Y intercept = 0.255 $R = 0.400$; R^2 (in %) = 15.99 Partial r for Y4 X5 = 0.307; $t = 4.999$; $P < .001$ Partial r for Y4 X7 = 0.134; $t = 2.098$; $P < .05$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y5 Laws and Generalizations	X5 Reasoning -- Number Relations	0.309	0.035	0.008	4.512	<.001
	X6 Verbal Meaning -- Synonyms	0.257	0.040	0.011	3.761	<.001
	Y intercept = 0.151					
	R = 0.504; R^2 (in %) = 25.43					
Y6 Total Topic: Number, Operations, Assumptions	Partial r for Y5 X5 = 0.279; $t = 4.512$; $P < .001$					
	Partial r for Y5 X6 = 0.236; $t = 3.761$; $P < .001$					
	X3 Associative Memory	0.104	0.079	0.039	1.999	<.05
	X5 Reasoning -- Number Relations	0.406	0.159	0.025	6.468	<.001
Y6 Total Topic: Number, Operations, Assumptions	X6 Verbal Meaning -- Synonyms	0.255	0.138	0.034	4.128	<.001
	Y intercept = 5.939					
	R = 0.629; R^2 (in %) = 39.59					
	Partial r for Y6 X3 = 0.128; $t = 0.020$; $P < .05$					
Y6 Total Topic: Number, Operations, Assumptions	Partial r for Y6 X5 = 0.385; $t = 6.468$; $P < .001$					
	Partial r for Y6 X6 = 0.258; $t = 4.128$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_1	b_1	S_{b_1}	t for b_1	P
Y7 Definition of Terms	The reliability of this subtopic was insufficient to warrant its inclusion in the analyses.					
Y8 Recognition of Geometric Figures	X6 Verbal Meaning -- Synonyms Y intercept = 1.949 $R = 0.349$; R^2 (in %) = 12.18 Partial r for Y8 X6 = 0.349; $t = 5.794$; $P < .001$	0.349	0.059	0.010	5.794	<.001
Y9 Measurement of Geometric Figures	Since this subtopic was not taught at this grade level, it was not included in the analyses.					
Y10 Total: Geometry	X1 Perceptual Discrimination -- Sets of Digits X2 Memory Span X6 Verbal Meaning -- Synonyms Y intercept = 3.568 $R = 0.356$; R^2 (in %) = 12.66 Partial r for Y10 X1 = 0.170; $t = 2.668$; $P < .01$ Partial r for Y10 X2 = -0.137; $t = 2.143$; $P < .01$ Partial r for Y10 X6 = 0.300; $t = 4.863$; $P < .001$	0.167 -0.134 0.298	0.062 -0.136 0.076	0.023 0.064 0.016	2.668 2.143 4.863	<.01 <.05 <.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y13 Maps, Graphs, etc.	X5 Reasoning -- Number Relations	0.176	0.022	0.009	2.379	
	X6 Verbal Meaning -- Synonyms	0.231	0.040	0.013	3.125	<.01
	Y Intercept = 1.303 $R = 0.362$; R^2 (in %) = 13.14 Partial r for Y13 X5 = 0.152; $t = 2.379$; $P < .05$ Partial r for Y13 X6 = 0.197; $t = 3.125$; $P < .01$					
Y14 Functions	The reliability of this subtopic was insufficient to warrant its inclusion in the analyses					
Y15 Total: Relations	X3 Associative Memory	0.122	0.076	0.032	2.349	<.05
	X5 Reasoning -- Number Relations	0.266	0.086	0.020	4.267	<.001
	X6 Verbal Meaning -- Synonyms	0.397	0.177	0.027	6.452	<.001
	Y Intercept = 3.001 $R = 0.635$; R^2 (in %) = 40.37 Partial r for Y15 X3 = 0.150; $t = 2.349$; $P < .05$ Partial r for Y15 X5 = 0.266; $t = 4.267$; $P < .001$ Partial r for Y15 X6 = 0.385; $t = 6.452$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
	X5 Reasoning -- Number Relations	0.564	0.072	0.007	10.236	<.001

Y18 Reading, Writing Using Numbers

$$Y_{\text{intercept}} = 0.769$$
$$\dot{R} = 0.561; R^2 \text{ (in \%)} = 31.46$$

Partial r for Y18 X5 = .561; $t = 10.286$; $P < .001$

Y19 Roman Numerals	X4	Associative Memory plus Conversion	0.138	0.028	0.012	2.306	<.05
	X2	Memory Span	0.120	0.093	0.044	2.107	<.05
	X5	Reasoning -- Number Relations	0.260	0.036	0.010	3.619	<.001
	X6	Verbal Meaning -- Synonyms	0.241	0.046	0.013	3.502	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y19 Roman Numerals (Continued)						
	Y intercept = -0.618					
	$R = 0.535$ R^2 (in %) = 28.59					
	Partial r for Y19 X4 = 0.148; $t = 2.306$; $P < .05$					
	Partial r for Y19 X2 = 0.135; $t = 2.107$; $P < .05$					
	Partial r for Y19 X5 = 0.228; $t = 3.619$; $P < .001$					
	Partial r for Y19 X6 = 0.221; $t = 3.502$; $P < .001$					
Y20 Total: Numeration						
	X4 Associative Memory plus Conversion	0.170	0.097	0.030	3.267	<.001
	X1 Perceptual Discrimination -- Sets of Digits	0.138	0.106	0.038	2.813	<.01
	X5 Reasoning -- Number Relations	0.254	0.099	0.022	4.415	<.001
	X6 Verbal Meaning -- Synonyms	0.309	0.166	0.032	5.214	<.001
	X8 Word Grouping	0.155	0.127	0.047	2.698	<.01

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y intercept = -1.603 $R = 0.742$; R^2 (in %) = 54.99 Partial r for Y20 X4 = 0.207; $t = 3.267$; $P < .001$ Partial r for Y20 X1 = 0.179; $t = 2.813$; $P < .01$ Partial r for Y20 X5 = 0.275; $t = 4.415$; $P < .001$ Partial r for Y20 X6 = 0.320; $t = 5.214$; $P < .001$ Partial r for Y20 X8 = 0.172; $t = 2.698$; $P < .01$						
Y20 Total: Numeration (Continued)						
X4 Associative Memory Plus Conversion X2 Memory Span X5 Reasoning -- Number Relations X6 Verbal Meaning -- Synonyms						
		0.124	0.025	0.013	1.972	<.05
		0.145	0.111	0.046	2.428	<.05
		0.228	0.031	0.010	3.024	<.01
		0.183	0.035	0.014	2.539	<.05
Y21 Concepts of Measurement						

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y21 Concepts of Measurement (Continued)	Y intercept = -0.664					
	R = 0.462; R^2 (in %) = 21.34					
	Partial r for Y21 X4 = 0.127; $t = 1.972$; $P < .05$					
	Partial r for Y21 X2 = 0.155; $t = 2.428$; $P < .05$					
	Partial r for Y21 X5 = 0.192; $t = 3.024$; $P < .01$					
	Partial r for Y21 X6 = 0.162; $t = 2.539$; $P < .05$					
Y22 Conversion of Units	X5 Reasoning -- Number Relations	0.337	0.041	0.008	4.959	<.001
	X6 Verbal Meaning -- Synonyms	0.239	0.040	0.011	3.519	<.001
	Y intercept = 0.479					
	R = 0.514; R^2 (in %) = 26.47					
	Partial r for Y22 X5 = 0.304; $t = 4.959$; $P < .001$					
	Partial r for Y22 X6 = 0.221; $t = 3.519$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	$\bar{t}$ for b_i	P
Y23 Operations - No Conversion of Units	X4 Associative Memory plus Conversion	0.209	0.037	0.010	3.799	<.001
	X2 Memory Span	0.120	0.081	0.035	2.299	<.05
	X5 Reasoning -- Number Relations	0.243	0.030	0.008	3.598	<.001
	X6 Verbal Meaning -- Synonyms	0.194	0.033	0.012	2.804	<.01
	X8 Word Grouping	0.165	0.042	0.017	2.500	<.01
Y intercept = -0.020						
$R = 0.641$; R^2 (in %) = 41.03						
Partial r for Y23 X4 = 0.239; $\bar{t} = 3.799$; $P < .001$						
Partial r for Y23 X2 = 0.147; $\bar{t} = 2.299$; $P < .05$						
Partial r for Y23 X5 = 0.227; $\bar{t} = 3.598$; $P < .001$						
Partial r for Y23 X6 = 0.179; $\bar{t} = 2.804$; $P < .01$						
Partial r for Y23 X3 = 0.160; $\bar{t} = 2.500$; $P < .01$						
Y24 Operations- Conversion of Units	X2 Memory Span	0.120	0.086	0.042	2.072	<.05
	X5 Reasoning -- Number Relations	0.222	0.029	0.009	3.159	<.01
	X6 Verbal Meaning -- Synonyms	0.299	0.053	0.013	4.208	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y26 Concepts of Fractions	X1 Perceptual Discrimination -- Sets of Digits	0.157	0.038	0.015	2.601	<.01
	X2 Memory Span	0.163	0.111	0.041	2.724	<.01
	X3 Associative Memory	0.121	0.028	0.014	2.025	<.05
	X6 Verbal Meaning -- Synonyms	0.278	0.047	0.010	4.636	<.001
Y intercept = -0.740 $R = 0.447$; R^2 (in %) = 19.96 Partial r for Y26 X1 = 0.166; $t = 2.601$; $P < .01$ Partial r for Y26 X2 = 0.174; $t = 2.724$; $P < .01$ Partial r for Y26 X3 = 0.130; $t = 2.025$; $P < .05$ Partial r for Y26 X6 = 0.287; $t = 4.636$; $P < .001$						
Y27 Addition of Fractions	The reliability of this subtopic was insufficient to warrant its inclusion in the analyses.					
	X4 Associative Memory Plus Conversion	0.257	0.045	0.010	4.456	<.001
Y28 Subtraction of Fractions	X5 Reasoning -- Number Relations	0.178	0.021	0.008	2.595	<.01
	X6 Verbal Meaning -- Synonyms	0.290	0.047	0.011	4.398	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	$\underline{t}$ for b_i	P
	Y intercept = -0.383					
	$R = 0.559$; R^2 (in %) = 31.24					
Y28 Subtraction of Fractions (Continued)	Partial r for Y28 X4 = 0.276; $\underline{t} = 4.456$; $P < .001$					
	Partial r for Y28 X5 = 0.165; $\underline{t} = 2.595$; $P < .01$					
	Partial r for Y28 X6 = 0.273; $\underline{t} = 4.398$; $P < .001$					
Y29 Multiplication of Fractions	Since this subtopic was not taught at this grade level, it was not included in the analyses.					
Y30 Division of Fractions	Since this subtopic was not taught at this grade level, it was not included in the analyses.					
Y31 Total Topic: Fractions						
	X4 Associative Memory Plus Conversion	0.162	0.062	0.021	2.984	<.01
	X2 Memory Span	0.174	0.252	0.073	3.463	<.001
	X5 Reasoning -- Number Relations	0.212	0.055	0.017	3.330	<.001
	X6 Verbal Meaning -- Synonyms	0.356	0.128	0.022	5.854	<.001
	X3 Associative Memory	0.113	0.056	0.026	2.194	<.05

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y31 Total Topic: Fractions (Continued)						
	Y intercept = -1.62					
	$R = 0.669$; R^2 (in %) = 44.74					
	Partial r for Y31 X4 = 0.190; $t = 2.984$; $P < .01$					
	Partial r for Y31 X2 = 0.220; $t = 3.463$; $P < .001$					
	Partial r for Y31 X5 = 0.211; $t = 3.330$; $P < .001$					
	Partial r for Y31 X6 = 0.355; $t = 5.854$; $P < .001$					
	Partial r for Y31 X3 = 0.141; $t = 2.194$; $P < .05$					

Results of the Step-wise Regression Analysis: Grade 5

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y1 Addition	X4 Associative Memory plus Conversion	0.307	0.048	0.009	5.187	<.001
	Y intercept = 2.525					
	$R = 0.307$; R^2 (in %) = 09.41					
	Partial r for Y1 $X_4 = 0.307$; $t = 5.187$; $P < .001$					
Y2 Subtraction	X5 Reasoning -- Number Relations	0.336	0.032	0.006	5.276	<.001
	X5 Verbal Meaning -- Synonyms	0.245	0.035	0.009	3.846	<.001
	Y intercept = 1.782					
	$R = 0.512$; R^2 (in %) = 26.18					
	Partial r for Y2 $X_5 = 0.312$; $t = 5.276$; $P < .001$					
	Partial r for Y2 $X_5 = 0.233$; $t = 3.846$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y3 Multiplication	X4 Associative Memory plus Conversion	0.143	0.030	0.012	2.540	<.05
	X5 Reasoning -- Number Relations	0.357	0.043	0.008	5.584	<.001
	X6 Word Grouping	0.222	0.060	0.018	3.423	<.001
Y intercept = 0.011 $R = 0.602$; R^2 (in %) = 36.21 Partial r for Y3 X4 = 0.157; $t = 2.540$; $P < .05$ Partial r for Y3 X5 = 0.329; $t = 5.584$; $P < .001$ Partial r for Y3 X6 = 0.209; $t = 3.423$; $P < .001$						
Y4 Division	X5 Reasoning -- Number Relations	0.379	0.044	0.008	5.761	<.001
	X6 Word Grouping	0.224	0.058	0.017	3.401	<.001
	Y intercept = -0.085 $R = 0.545$; R^2 (in %) = 29.67 Partial r for Y4 X5 = 0.338; $t = 5.761$; $P < .001$ Partial r for Y4 X6 = 0.207; $t = 3.401$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y5 Laws and Generalizations	X5 Reasoning --- Number Relations	0.216	0.021	0.007	2.983	<.01
	X6 Verbal Meaning -- Synonyms	0.186	0.028	0.011	2.620	<.01
	X8 Word Grouping	0.157	0.035	0.017	2.087	<.05
	Y intercept = 0.958 $R = 0.474$ R^2 (in %) = 22.45 Partial r for Y5 X5 = 0.183; $t = 2.983$; P <.01 Partial r for Y5 X6 = 0.161; $t = 2.620$; P <.01 Partial r for Y5 X8 = 0.129; $t = 2.088$; P <.05					
Y6 Total Topic: Number, Operations, Assumptions	X4 Associative Memory plus Conversion	0.171	0.102	0.028	3.636	<.001
	X5 Reasoning -- Number Relations	0.374	0.129	0.019	6.708	<.001
	X6 Verbal Meaning -- Synonyms	0.183	0.097	0.029	3.387	<.001
	X8 Word Grouping	0.204	0.158	0.045	3.476	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y6 Total Topic: Number, Operations, Assumptions (Continued)	Y intercept = 4.300					
	$R = 0.745$; R^2 (in %) = 55.48					
	Partial r for Y6 X4 = 0.222; $t = 3.636$; $P < .001$					
	Partial r for Y6 X5 = 0.387; $t = 6.708$; $P < .001$					
	Partial r for Y6 X6 = 0.207; $t = 3.387$; $P < .001$					
	Partial r for Y6 X8 = 0.212; $t = 3.476$; $P < .001$					
Y7 Definition of Terms	The reliability of this subtopic was insufficient to warrant inclusion in the analyses.					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	$\underline{t}$ for b_i	P
Y8 Recognition of Geometric Figures	X2 Memory Span	-0.107	-0.067	0.033	1.997	<.05
	X5 Reasoning -- Number Relations	0.278	0.030	0.007	4.178	<.001
	X6 Verbal Meaning -- Synonyms	0.218	0.036	0.011	3.398	<.001
	X7 Spatial Relations -- Visualization	0.146	0.035	0.014	2.445	<.05
Y intercept = 1.457 $R = 0.528$; R^2 (in %) = 27.87 Partial r for Y8 X2 = -0.124; $\underline{t} = 1.997$; $P < .05$ Partial r for Y8 X5 = 0.253; $\underline{t} = 4.178$; $P < .001$ Partial r for Y8 X6 = 0.208; $\underline{t} = 3.398$; $P < .001$ Partial r for Y8 X7 = 0.151; $\underline{t} = 2.445$; $P < .05$						
Y9 Measurement of Geometric Figures	X5 Reasoning -- Number Relations	0.320	0.037	0.008	4.767	<.001
	X8 Word Grouping	0.258	0.067	0.017	3.840	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y11 Equality	X5 Reasoning -- Number Relations	0.322	0.034	0.007	4.784	<.001
	X6 Verbal Meaning -- Synonyms	0.192	0.031	0.011	2.910	<.01
	X8 Word Grouping	0.155	0.037	0.016	2.216	<.05
Y intercept = 0.910 $R = 0.572$; R^2 (in %) = 32.74 Partial r for Y11 X5 = 0.286; $t = 4.784$; $P < .001$ Partial r for Y11 X6 = 0.179; $t = 2.910$; $P < .01$ Partial r for Y11 X8 = 0.137; $t = 2.216$; $P < .05$						
Y12 Order Relations	X5 Reasoning -- Number Relations	0.368	0.043	0.007	6.409	<.001
	X6 Verbal Meaning -- Synonyms	0.353	0.063	0.010	6.154	<.001
	Y intercept = *0.629 $R = 0.633$; R^2 (in %) = 40.05 Partial r for Y12 X5 = 0.371; $t = 6.409$; $P < .001$ Partial r for Y12 X6 = 0.358; $t = 6.154$; $P < .001$					

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Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y17 Number Bases	X5 Reasoning -- Number Relations	0.395	0.055	0.009	6.322	<.001
	X6 Verbal Meaning -- Synonyms	0.209	0.045	0.014	3.351	<.001
	Y intercept = -0.349					
	R = 0.538; R^2 (in %) = 28.93					
Y18 Reading, Writing, Using Numbers	Partial r for Y17 X5 = 0.366; t 6.322; P <.001					
	Partial r for Y17 X6 = 0.204; t 3.351; P <.001					
	X1 Perceptual Discrimination -- sets of Digits	0.134	0.031	0.011	2.785	<.01
	X5 Reasoning -- Number Relations	0.278	0.031	0.007	4.432	<.001
	X6 Verbal Meaning -- Synonyms	0.266	0.046	0.011	4.315	<.001
	X8 Word Grouping	0.182	0.046	0.016	2.781	<.01

Y intercept = -0.407

R = 0.647; R^2 (in %) = 41.79

Partial r for Y18 X1 = 0.172; $t = 2.785$; $P < .01$

Partial r for Y18 X5 = 0.267; $t = 4.432$; $P < .001$

Partial r for Y18 X6 = 0.260; $t = 4.315$; $P < .001$

Partial r for Y18 X8 = 0.171; $t = 2.781$; $P < .01$

Y18 Reading, Writing,
Using Numbers
(Continued)

X5 Reasoning -- Number Relations	0.331	0.044	0.008	5.451	<.001
X6 Verbal Meaning -- Synonyms	0.320	0.065	0.012	5.260	<.001

Y19 Roman Numerals

Y intercept = -0.051

R = 0.572; R^2 (in %) 32.69

Partial r for Y19 X5 = 0.321; $t = 5.451$; $P < .001$

Partial r for Y19 X6 = 0.311; $t = 5.260$; $P < .001$

X5 Reasoning -- Number Relations	0.368	0.131	0.020	6.680	<.001
X6 Verbal Meaning -- Synonyms	0.308	0.169	0.030	5.694	<.001
X8 Word Grouping	0.195	0.156	0.046	3.406	<.001

Y20 Total: Numeration

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y20 Total: Numeration (Continued)						
	Y intercept = -0.541					
	$R = 0.741$; R^2 (in %) = 54.92					
	Partial r for Y20 X5 = 0.385; $t = 6.680$; $P < .001$					
	Partial r for Y20 X6 = 0.335; $t = 5.694$; $P < .001$					
	Partial r for Y20 X8 = 0.208; $t = 3.406$; $P < .001$					
Y21 Concepts of Measurement						
X5	Reasoning -- Number Relations	0.181	0.022	0.009	2.568	<.05
X6	Verbal Meaning -- Synonyms	0.179	0.034	0.013	3.636	<.01
X7	Spatial Relations -- Visualization	0.146	0.040	0.016	2.458	<.05
X8	Word Grouping	0.185	0.051	0.020	2.555	<.05
Y intercept = -0.655						
	$R = 0.547$; R^2 (in %) = 29.91					
	Partial r for Y21 X5 = 0.159; $t = 2.568$; $P < .05$					
	Partial r for Y21 X6 = 0.163; $t = 2.636$; $P < .01$					
	Partial r for Y21 X7 = 0.152; $t = 2.458$; $P < .05$					
	Partial r for Y21 X8 = 0.158; $t = 2.555$; $P < .05$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y22 Conversion of Units	X5 Reasoning -- Number Relations	0.315	0.035	0.007	4.875	<.001
	X6 Verbal Meaning -- Synonyms	0.259	0.045	0.011	4.085	<.001
	X8 Word Grouping	0.148	0.037	0.017	2.201	<.05
Y intercept = -0.213						
R = 0.616; R^2 (in %) = 37.90						
Partial r for Y22 X5 = 0.291; t 4.875; P <.001						
Partial r for Y22 X6 = 0.247; t = 4.085; P <.001						
Partial r for Y22 X8 = 0.136; t = 2.201; P <.05						
Y23 Operations - No Conversion of Units	X5 Reasoning -- Number Relations	0.214	0.022	0.007	3.277	<.001
	X6 Verbal Meaning -- Synonyms	0.240	0.038	0.010	3.737	<.001
	X8 Word Grouping	0.255	0.059	0.016	3.754	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y23 Operations - No Conversion of Units (Continued)	Y intercept = 1.022					
	$R = 0.603$; R^2 (in %) = 36.37					
	Partial r for Y23 X5 = 0.200; $t = 3.277$; $P < .001$					
	Partial r for Y23 X6 = 0.227; $t = 3.737$; $P < .001$					
	Partial r for Y23 X8 = 0.228; $t = 3.754$; $P < .001$					
Y24 Operations - Conversion of Units	X5 Reasoning -- Number Relations	0.466	0.056	0.007	8.207	<.001
	X7 Spatial Relations -- Visualization	0.181	0.049	0.015	3.177	<.01
	Y intercept = 0.109					
	$R = 0.568$; R^2 (in %) = 32.26					
	Partial r for Y24 X5 = 0.455; $t = 8.207$; $P < .001$					
	Partial r for Y24 X7 = 0.194; $t = 3.177$; $P < .01$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y25 Total: Measurement	X5 Reasoning -- Number Relations	0.364	0.125	0.019	6.703	<.001
	X6 Verbal Meaning -- Synonyms	0.231	0.123	0.028	4.418	<.001
	X7 Spatial Relations -- Visualization	0.169	0.130	0.035	3.680	<.001
	X8 Word Grouping	0.191	0.147	0.043	3.427	<.001
Y intercept = 0.405 $R = 0.765$; R^2 (in %) = 58.51 Partial r for Y25 X5 = 0.386; $t = 6.703$; $P < .001$ Partial r for Y25 X6 = 0.266; $t = 4.418$; $P < .001$ Partial r for Y25 X7 = 0.224; $t = 3.680$; $P < .001$ Partial r for Y25 X8 = 0.209; $t = 3.427$; $P < .001$						
Y26 Concepts of Fractions	X2 Memory Span	-0.158	-0.097	0.034	2.827	<.01
	X5 Reasoning -- Number Relations	0.436	0.046	0.006	7.819	<.001
	Y intercept = 1.466 $R = 0.452$; R^2 (in %) = 20.40 Partial r for Y26 X2 = 0.173; $t = 2.827$; $P < .01$ Partial r for Y26 X5 = 0.438; $t = 7.819$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y27 Addition of Fractions	X5 Reasoning -- Number Relations	0.402	0.042	0.006	6.708	<.001
	X8 Word Grouping	0.316	0.074	0.014	5.263	<.01
	Y intercept = 0.234					
	R = 0.645; R^2 (in %) = 41.57					
Y28 Subtraction of Fractions	Partial r for Y27 X5 = 0.385; $t = 6.708$; P <.001					
	Partial r for Y27 X8 = 0.311; $t = 5.263$; P <.01					
Y28 Subtraction of Fractions	X5 Reasoning -- Number Relations	.623	0.071	0.006	11.900	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y28 Subtraction of Fractions (Continued)	Y intercept = 0.681 R = .623; R^2 (in %) = 38.77 Partial r for Y28 X5 = .623; $t = 11.900$; $P < .001$					
Y29 Multiplication of Fractions	Since this subtopic was not taught at this grade level, it was excluded from the analyses.					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y30 Division of Fractions						
Since this subtopic was not taught at this grade level, it was excluded from the analyses.						
Y31 Total Topic: Fractions						
X4	Associative Memory plus Conversion	0.098	0.044	0.021	2.083	<.05
X2	Memory Span	-0.093	-0.139	0.063	2.223	<.05
X5	Reasoning -- Number Relations	0.445	0.115	0.015	7.895	<.001
X6	Verbal Meaning -- Synonyms	0.136	0.539	0.021	2.543	<.05
X7	Spatial Relations -- Visualization	0.137	0.797	0.027	2.902	<.01
X8	Word Grouping	0.136	0.785	0.034	2.329	<.05

Mathematics Variable	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
	Y intercept = 0.830					
	$R = 0.757$; R^2 (in %) = 57.26					
	Partial r for Y31 X4 = 0.130; $t = 2.083$; $P < .05$					
	Partial r for Y31 X2 = -0.138; $t = 2.223$; $P < .05$					
	Partial r for Y31 X5 = 0.444; $t = 7.895$; $P < .001$					
	Partial r for Y31 X6 = 0.158; $t = 2.543$; $P < .05$					
	Partial r for Y31 X7 = 0.179; $t = 2.902$; $P < .01$					
	Partial r for Y31 X8 = 0.145; $t = 2.329$; $P < .05$					

Y31 Total Topic:
Fractions (Continued)

Results of the Step-wise Regression Analysis: Grade 6

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_1	b_1	S_{b_1}	t for b_1	P
Y1 Addition	X4 Associative Memory Plus Conversion	0.168	0.014	0.005	2.630	<.01
	Y intercept = 3.019					
	$R = 0.168$; R^2 (in %) = 02.82					
	Partial r for Y1 X4 = 0.168; $t = 2.630$; $P < .01$					
Y2 Subtraction	X1 Perceptual Discrimination Sets of Digits	0.162	0.033	0.012	2.710	<.01
	X6 Verbal Meaning -- Synonyms	0.371	0.058	0.009	6.198	<.001
	Y intercept = 1.627					
	$R = 0.433$; R^2 (in %) = 18.76					
	Partial r for Y2 X1 = 0.173; $t = 2.710$; $P < .01$					
	Partial r for Y2 X6 = 0.374; $t = 6.199$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	$\underline{t}$ for b_i	P
Y3 Multiplication	X1 Perceptual Discrimination -- Sets of Digits	0.120	0.030	0.014	2.168	<.05
	X3 Associative Memory	0.173	0.040	0.014	2.820	<.01
	X5 Reasoning -- Number Relations	0.207	0.026	0.008	3.111	<.01
	X6 Verbal Meaning -- Synonyms	0.172	0.034	0.014	2.350	<.05
	X3 Word Grouping	0.156	0.046	0.021	2.147	<.05
	Y intercept = -0.115 R = 0.604; R^2 (in %) = 36.47 Partial r for Y3X1 = 0.140; $\underline{t}$ = 2.168; P <.05 Partial r for Y3 X3 = 0.181; $\underline{t}$ = 2.820; P <.01 Partial r for Y3 X5 = 0.199; $\underline{t}$ = 3.111; P <.01 Partial r for Y3 X6 = 0.152; $\underline{t}$ = 2.350; P <.05 Partial r for Y3 X8 = 0.139; $\underline{t}$ = 2.147; P <.05					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_1	b_1	S_{b_1}	t for b_1	P
Y4 Division	X4 Associative Memory plus Conversion	0.209	0.022	0.007	3.169	<.01
	X1 Perceptual Discrimination -- Sets of Digits	0.203	0.056	0.017	3.361	<.001
	X6 Verbal Meaning -- Synonyms	0.256	0.055	0.013	4.077	<.001
	Y intercept = 0.238 R = 0.498; R^2 (in %) = 24.80 Partial r for Y4 X4 = 0.202; $t = 3.169$; $P < .01$ Partial r for Y4 X1 = 0.214; $t = 3.361$; $P < .001$ Partial r for Y4 X6 = 0.257; $t = 4.077$; $P < .001$					
Y5 Laws and Generalizations	X5 Reasoning -- Number Relations	0.234	0.023	0.007	3.303	<.001
	X6 Verbal Meaning -- Synonyms	0.167	0.025	0.011	2.356	<.05
	Y intercept = 1.677 R = 0.350; R^2 (in %) = 12.24 Partial r for Y5 X5 = 0.210; $t = 3.303$; $P < .001$ Partial r for Y5 X6 = 0.151; $t = 2.356$; $P < .05$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	b _i	S _{b_i}	t for b _i	P
X4 Associative Memory plus Conversion		0.130	0.036	0.018	2.086	<.05
X1 Perceptual Discrimination -- Sets of Digits		0.154	0.114	0.041	2.765	<.01
X5 Reasoning -- Number Relations		0.192	0.071	0.024	2.925	<.01
X6 Verbal Meaning -- Synonyms		0.220	0.126	0.041	3.044	<.01
X8 Word Grouping		0.148	0.127	0.062	2.063	<.05
Y6 Total Topic: Number, Operations, Assumptions						
Y intercept = 5.823						
$R = 0.616; R^2 \text{ (in \%)} = 37.97$						
Partial r for Y6 X4 = 0.135; $\underline{t} = 2.086; P < .05$						
Partial r for Y6 X1 = 0.178; $\underline{t} = 2.765; P < .01$						
Partial r for Y6 X5 = 0.188; $\underline{t} = 2.925; P < .01$						
Partial r for Y6 X6 = 0.195; $\underline{t} = 3.044; P < .01$						
Partial r for Y6 X8 = 0.134; $\underline{t} = 2.063; P < .05$						

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	$\underline{t}$ for b_i	P
Y7 Definition of Terms	X5 Reasoning -- Number Relations	0.236	0.030	0.009	3.355	<.001
	X6 Verbal Meaning -- Synonyms	0.180	0.035	0.014	2.551	<.05
	Y intercept = 0.257					
	$R = 0.363$; R^2 (in %) = 13.16					
	Partial r for Y7 X5 = 0.213; $\underline{t} = 3.355$; $P < .001$					
Y8 Recognition of Geometric Figures	Partial r for Y7 X6 = 0.164; $\underline{t} = 2.551$; $P < .05$					
	X6 Verbal Meaning -- Synonyms	0.373	0.068	0.013	5.169	<.001
	X7 Spatial Relations -- Visualization	0.139	0.030	0.013	2.342	<.05
	X8 Word Grouping	0.155	0.042	0.020	2.065	<.05
	Y intercept = 0.356					
	$R = 0.553$; R^2 (in %) = 30.62					
	Partial r for Y8 X6 = 0.319; $\underline{t} = 5.169$; $P < .001$					
	Partial r for Y8 X7 = 0.151; $\underline{t} = 2.342$; $P < .05$					
	Partial r for Y8 X8 = 0.133; $\underline{t} = 2.065$; $P < .05$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y9 Measurement of Geometric Figures	X4 Associative Memory plus Conversion	0.120	0.013	0.006	2.076	<.05
	X5 Reasoning -- Number Relations	0.255	0.036	0.009	4.049	<.001
	X6 Verbal Meaning -- Synonyms	0.179	0.040	0.015	2.584	<.01
	X7 Spatial Relations -- Visualization	0.113	0.030	0.014	2.085	<.05
	X8 Word Grouping	0.196	0.065	0.024	2.754	<.01

Y intercept = -1.108

$R = 0.653$; R^2 (in %) = 42.70

Partial r for Y9 X4 = 0.134; $t = 2.076$; $P < .05$

Partial r for Y9 X5 = 0.256; $t = 4.049$; $P < .001$

Partial r for Y9 X6 = 0.167; $t = 2.584$; $P < .01$

Partial r for Y9 X7 = 0.135; $t = 2.085$; $P < .05$

Partial r for Y9 X8 = 0.177; $t = 2.754$; $P < .01$

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B _i	b _i	S _{b_i}	t for b _i	p
X4 Associative Memory plus Conversion		0.143	0.032	0.012	2.640	<.01
X5 Reasoning -- Number Relations		0.227	0.068	0.017	3.844	<.001
X6 Verbal Meaning -- Synonyms		0.261	0.118	0.029	4.018	<.001
X7 Spatial Relations -- Visualization		0.132	0.070	0.027	2.583	<.01
X8 Word Grouping		0.175	0.118	0.045	2.637	<.01
Y10 Total: Geometry						
Y intercept = -0.626						
R = 0.706; R ² (in %) = 49.79						
Partial r for Y10 X4 = 0.170; t = 2.640; P <.01						
Partial r for Y10 X5 = 0.244; t = 3.844; P <.001						
Partial r for Y10 X6 = 0.254; t = 4.018; P <.001						
Partial r for Y10 X7 = 0.167; t = 2.583; P <.01						
Partial r for Y10 X8 = 0.170; t = 2.637; P <.01						

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y11 Equality	X5 Reasoning -- Number Relations	0.323	0.036	0.007	4.878	<.001
	X7 Spatial Relations -- Visualization	0.121	0.025	0.013	1.975	<.05
	X8 Word Grouping	0.196	0.050	0.018	2.845	<.01
Y intercept = 0.887 $R = 0.516$; R^2 (in %) = 26.62 Partial r for Y11 X5 = 0.303; $t = 4.878$; $P < .001$ Partial r for Y11 X7 = 0.128; $t = 1.975$; $P < .05$ Partial r for Y11 X8 = 0.182; $t = 2.845$; $P < .01$						
Y12 Order Relations	X5 Reasoning -- Number Relations	0.301	0.047	0.010	4.897	<.001
	X6 Verbal Meaning -- Synonyms	0.274	0.066	0.017	3.960	<.001
	X8 Word Grouping	0.178	0.064	0.025	2.533	<.01

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y12 Order Relations (Continued)						
	Y intercept = -1.862					
	$R = 0.634$; R^2 (in %) = 40.17					
	Partial r for Y12 X5 = 0.304; $t = 4.897$; $P < .001$					
	Partial r for Y12 X6 = 0.250; $t = 3.960$; $P < .001$					
	Partial r for Y12 X8 = 0.163; $t = 2.533$; $P < .05$					
Y13 Maps, Graphs, Charts						
	X5 Reasoning -- Number Relations	0.208	0.024	0.008	2.967	<.01
	X6 Verbal Meaning -- Synonyms	0.224	0.041	0.013	3.189	<.01
Y14 Functions						
	X1 Perceptual Discrimination -- Sets of Digits	0.169	0.049	0.016	2.992	<.01
	X5 Reasoning -- Number Relations	0.190	0.027	0.009	2.908	<.01
	X8 Word Grouping	0.356	0.119	0.021	5.555	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y14 Functions (Continued)						
	Y intercept = -1.132					
	$R = 0.553$; R^2 (in %) = 30.63					
	Partial r for Y14 X1 = 0.191; $t = 2.992$; $P < .01$					
	Partial r for Y14 X5 = 0.186; $t = 2.908$; $P < .01$					
	Partial r for Y14 X8 = 0.340; $t = 5.555$; $P < .001$					
Y15 Total: Relations						
X5 Reasoning -- Number Relations		0.340	0.129	0.021	6.181	<.001
X6 Verbal Meaning -- Synonyms		0.240	0.142	0.036	3.891	<.001
X7 Spatial Relations -- Visualization		0.113	0.079	0.034	2.285	<.05
X8 Word Grouping		0.221	0.195	0.057	3.442	<.001
Y15 Total: Relations						
	Y intercept = -1.029					
	$R = 0.726$; R^2 (in %) = 52.69					
	Partial r for Y15 X5 = 0.374; $t = 6.181$; $P < .001$					
	Partial r for Y15 X6 = 0.246; $t = 3.891$; $P < .001$					
	Partial r for Y15 X7 = 0.147; $t = 2.285$; $P < .05$					
	Partial r for Y15 X8 = 0.219; $t = 3.442$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y16 Sets	X5 Reasoning -- Number Relations	0.256	0.032	0.008	4.079	<.001
	Y intercept = 1.333					
	R = 0.256; R^2 (in %) = 6.53					
	Partial r for Y16 X5 = 0.256; $t = 4.079$; $P < .001$					
Y17 Number Bases	X5 Reasoning -- Number Relations	0.230	0.037	0.011	3.283	<.001
	X8 Word Grouping	0.236	0.088	0.026	3.374	<.001
	Y intercept = -1.260					
	R = 0.407; R^2 (in %) = 16.60					
Y18 Reading, Writing, Using Numbers	Partial r for Y17 X5 = 0.209; $t = 3.283$; $P < .001$					
	Partial r for Y17 X8 = 0.214; $t = 3.374$; $P < .001$					
	X5 Reasoning -- Number Relations	0.268	0.036	0.008	4.395	<.001
	X6 Verbal Meaning -- Synonyms	0.249	0.053	0.015	3.634	<.001
	X8 Word Grouping	0.244	0.077	0.022	3.515	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y18 Reading, Writing, Using Numbers (Continued)						
	Y intercept = -0.954					
	R = 0.641; R^2 (in %) = 41.13					
	Partial r for Y18 X5 = 0.275; t = 4.395; P < .001					
	Partial r for Y18 X6 = 0.230; t = 3.634; P < .001					
	Partial r for Y18 X8 = 0.223; t = 3.515; P < .001					
Y19 Roman Numerals						
	X3 Associative Memory	0.151	0.041	0.017	2.425	<.05
	X5 Reasoning -- Number Relations	0.219	0.032	0.010	3.207	<.01
	X8 Word Grouping	0.296	0.101	0.022	4.515	<.001
Y intercept = -0.743						
	R = 0.537; R^2 (in %) = 28.85					
	Partial r for Y19 X3 = 0.156; t = 2.425; P < .001					
	Partial r for Y19 X5 = 0.204; t = 3.207; P < .01					
	Partial r for Y19 X8 = 0.282; t = 4.515; P < .001					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y20 Total: Numeration	X5 Reasoning -- Number Relations	0.321	0.123	0.022	5.637	<.001
	X6 Verbal Meaning -- Synonyms	0.211	0.126	0.038	3.311	<.001
	X7 Spatial Relations -- Visualization	0.117	0.082	0.036	2.298	<.05
	X8 Word Grouping	0.238	0.211	0.059	3.570	<.001
Y intercept = -2.611						
$R = 0.702$; R^2 (in %) = 49.34						
Partial r for Y20 X5 = 0.345; $t = 5.637$; $P < .001$						
Partial r for Y20 X6 = 0.211; $t = 3.311$; $P < .001$						
Partial r for Y20 X7 = 0.148; $t = 2.298$; $P < .05$						
Partial r for Y20 X8 = 0.227; $t = 3.570$; $P < .001$						
Y21 Concepts of Measurement	X5 Reasoning -- Number Relations	0.150	0.022	0.010	2.274	<.05
	X6 Verbal Meaning -- Synonyms	0.278	0.063	0.017	3.739	<.001
	X8 Word Grouping	0.224	0.076	0.026	2.982	<.01

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y21 Concepts of Measurement (Continued)	Y intercept = -1.243					
	$R = 0.557$; R^2 (in %) = 31.02					
	Partial r for Y21 X5 = 0.146; $t = 2.274$; $P < .05$					
	Partial r for Y21 X6 = 0.237; $t = 3.739$; $P < .001$					
	Partial r for Y21 X8 = 0.191; $t = 2.982$; $P < .01$					
Y22 Conversion of Units	X3 Associative Memory	0.140	0.035	0.015	2.320	<.05
	X5 Reasoning -- Number Relations	0.307	0.041	0.009	4.811	<.001
	X6 Verbal Meaning -- Synonyms	0.283	0.059	0.013	4.495	<.001
	Y intercept = -0.189					
	$R = 0.595$; R^2 (in %) = 35.37					
	Partial r for Y22 X3 = 0.149; $t = 2.320$; $P < .05$					
	Partial r for Y22 X5 = 0.299; $t = 4.811$; $P < .001$					
	Partial r for Y22 X6 = 0.281; $t = 4.495$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y23 Operations - No Conversion of Units	X1 Perceptual Discrimination -- Sets of Digits	0.182	0.034	0.011	3.046	<.01
	X2 Memory Span	-0.130	-0.065	0.029	2.258	<.05
	X5 Reasoning -- Number Relations	0.250	0.024	0.006	3.706	<.001
	X6 Verbal Meaning -- Synonyms	0.217	0.032	0.010	3.283	<.001
Y intercept = 2.197 $R = 0.493$; R^2 (in Z) = 24.33 Partial r for Y23 X1 = 0.195; $t = 3.046$; $P < .01$ Partial r for Y23 X2 = -0.146; $t = 2.258$; $P < .05$ Partial r for Y23 X5 = 0.235; $t = 3.706$; $P < .001$ Partial r for Y23 X6 = 0.209; $t = 3.283$; $P < .001$						
Y24 Operations - Conversion of Units	X5 Reasoning -- Number Relations	0.283	0.038	0.009	4.460	<.001
	X6 Verbal Meaning -- Synonyms	0.249	0.052	0.015	3.485	<.001
	X8 Word Grouping	0.181	0.056	0.023	2.493	<.05

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y24 Operations - Conversion of Units (Continued)	Y intercept = -1.061 R = 0.600; R^2 (in %) = 36.05 Partial r for Y24 X5 = 0.279; $t = 4.460$; $P < .001$ Partial r for Y24 X6 = 0.221; $t = 3.485$; $P < .001$ Partial r for Y24 X8 = 0.160; $t = 2.493$; $P < .001$					
	X1 Perceptual Discrimination -- Sets of Digits	0.119	0.096	0.039	2.497	<.05
	X5 Reasoning -- Number Relations	0.286	0.115	0.023	5.049	<.001
	X6 Verbal Meaning -- Synonyms	0.296	0.185	0.039	4.711	<.001
	X8 Word Grouping	0.214	0.200	0.060	3.358	<.001
Y25 Total: Measurement	Y intercept = -1.822 R = 0.713; R^2 (in %) = 50.84 Partial r for Y25 X1 = 0.161; $t = 2.497$; $P < .05$ Partial r for Y25 X5 = 0.313; $t = 5.049$; $P < .001$ Partial r for Y25 X6 = 0.294; $t = 4.711$; $P < .001$ Partial r for Y25 X8 = 0.214; $t = 3.358$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y26 Concepts of Fractions	X2 Memory Span	-0.124	-0.079	0.033	2.430	<.05
	X5 Reasoning -- Number Relations	0.155	0.019	0.007	2.488	<.05
	X6 Verbal Meaning -- Synonyms	0.264	0.049	0.013	3.795	<.001
	X7 Spatial Relations -- Visualization	0.125	0.028	0.012	2.251	<.05
	X8 Word Grouping	0.246	0.069	0.020	3.391	<.001
Y intercept = 0.225 $R = 0.632$; R^2 (in %) = 39.96 Partial r for Y26 X2 = -0.157; $t = 2.430$; $P < .05$ Partial r for Y26 X5 = 0.161; $t = 2.488$; $P < .05$ Partial r for Y26 X6 = 0.241; $t = 3.795$; $P < .001$ Partial r for Y26 X7 = 0.146; $t = 2.251$; $P < .05$ Partial r for Y26 X8 = 0.216; $t = 3.391$; $P < .001$						
Y27 Addition of Fractions	X3 Associative Memory	0.120	0.026	0.013	2.019	<.05
	X5 Reasoning -- Number Relations	0.183	0.021	0.008	2.791	<.01
	X8 Word Grouping	0.408	0.111	0.017	6.491	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y27 Addition of Fractions (Continued)	Y intercept = 0.061					
	$R = 0.590$; R^2 (in %) = 34.75					
	Partial r for Y27 X2 = 0.130; $t = 2.019$; $P < .05$					
	Partial r for Y27 X5 = 0.179; $t = 2.791$; $P < .01$					
	Partial r for Y27 X8 = 0.389; $t = 6.491$; $P < .001$					
Y28 Subtraction of Fractions	X4 Associative Memory plus Conversion	0.126	0.012	0.006	2.049	<.05
	X5 Reasoning -- Number Relations	0.200	0.025	0.008	3.111	<.01
	X6 Verbal Meaning -- Synonyms	0.380	0.075	0.013	5.969	<.001
	Y intercept = 0.293					
	$R = 0.586$; R^2 (in %) = 34.34					
	Partial r for Y28 X4 = 0.132; $t = 2.049$; $P < .05$					
	Partial r for Y28 X5 = 0.199; $t = 3.111$; $P < .01$					
	Partial r for Y28 X6 = 0.362; $t = 5.969$; $P < .001$					

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y29 Multiplication of Fractions	X4 Associative Memory plus Conversion	0.193	0.020	0.006	3.362	<.001
	X6 Verbal Meaning -- Synonyms	0.267	0.056	0.015	3.810	<.001
	X7 Spatial Relations -- Visualization	0.121	0.030	0.014	2.173	<.05
	X8 Word Grouping	0.234	0.073	0.022	3.305	<.001
	Y intercept = -0.820 $R = 0.627$; R^2 (in %) = 39.27 Partial r for Y29 X4 = 0.214; $t = 3.362$; $P < .001$ Partial r for Y29 X6 = 0.241; $t = 3.810$; $P < .001$ Partial r for Y29 X7 = 0.140; $t = 2.173$; $P < .05$ Partial r for Y29 X8 = 0.211; $t = 3.305$; $P < .001$					
Y30 Division of Fractions	X4 Associative Memory plus Conversion	0.202	0.020	0.006	3.151	<.01
	X6 Verbal Meaning -- Synonyms	0.331	0.068	0.013	5.158	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y3C Division of Fractions (Continued)						
Y intercept = 0.076						
$R = 0.458$; R^2 (in Z) = 20.93						
Partial r for Y30 $X_4 = 0.201$; $t = 3.151$; $P < .01$						
Partial r for Y30 $X_6 = 0.318$; $t = 5.158$; $P < .001$						
Y3I Operations with Fractions						
X4 Associative Memory plus Conversion						
		0.174	0.063	0.018	3.489	<.001
X5 Reasoning -- Number Relations						
		0.158	0.075	0.026	2.911	<.01
X6 Verbal Meaning -- Synonyms						
		0.331	0.244	0.044	5.548	<.001
X7 Spatial Relations -- Visualization						
		0.103	0.090	0.041	2.198	<.05
X8 Word Grouping						
		0.223	0.245	0.067	3.647	<.001

Mathematics Variable Number and Name	Intellectual Processes Variable Number and Name	B_i	b_i	S_{b_i}	t for b_i	P
Y intercept = -1.114 $R = 0.759$; R^2 (in %) = 57.54 Partial t for Y31 X4 = 0.222; $t = 3.489$; $P < .001$ Partial t for Y31 X5 = 0.187; $t = 2.911$; $P < .01$ Partial t for Y31 X6 = 0.341; $t = 5.548$; $P < .001$ Partial t for Y31 X7 = 0.142; $t = 2.198$; $P < .05$ Partial t for Y31 X8 = 0.232; $t = 3.647$; $P < .001$						
Y31 Total Topic: Fractions (Continued)						